Contagion of Financial Crises in Sovereign Debt Markets

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Abstract

This paper develops a quantitative model of debt, default, and contagion of financial crises for small open economies that interact with risk averse international investors. The paper extends the recent literature on endogenous default risk to the case in which several emerging economies that cannot credibly commit to honor their international debts have common investors. The existence of common investors with preferences that exhibit decreasing absolute risk aversion generates financial links between the emerging economies sovereign debt markets that help to explain the endogenous determination of credit limits, capital flows, and the risk premium in sovereign bond prices as function not only of the economy’s fundamentals, the investors’ characteristics (wealth, and degree of risk aversion) but more importantly of the fundamentals of other emerging economies. Therefore this paper provides a theoretical formalization that is the base for and endogenous explanation of the contagion of financial crises.

Keywords: Contagion; Sovereign Debt; Financial Links; Default
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1 Introduction

The last two decades of the 20th century can be described as a period of turbulence in international capital markets. During this period the world has witnessed many currency and financial crises. These crises can be characterized by the following stylized facts: a) the sudden loss of access to international capital markets, b) large reversal of the current account deficit, c) a collapse of domestic production and aggregate demand, and d) the simultaneous occurrence of these crises across countries. The simultaneity of crises can be seen in Figure 1, which shows spikes in sovereign bond spreads across several Latin American countries in the aftermath of the Mexican Crisis of 1994 and the Russian Crisis of 1998. The current paper is concerned with understanding why simultaneous financial crises occur and whether contagion spreads crises.

In the existing literature, the simultaneity of financial crises across economies can be explained by three different mechanisms, some of which are termed contagion and some of which are not: First, crises can be simultaneous when emerging economies are affected by a common shock, e.g., a shock to international interest rates, a shock to terms of trade, etc. Second, crises can be simultaneous when there exist fundamental links across emerging economies which facilitate the transmission of shocks from a ground zero country to other emerging economies. Examples of these types of links include financial links between countries that share investors and trade links between economies that are commercial partners or competitors. Third, crises might be simultaneous as a consequence of exogenous changes in investors’ perceptions: when a crisis hits one country, the ground-zero country, investors’ may perceive a higher default probability in non ground-zero countries. As a consequence, lenders will reduce funds to non ground-zero countries and crisis spreads. While there is debate in the literature about which of these three mechanisms should properly be called contagion, the current paper defines contagion as the transmission of negative productivity shocks that cannot be explained by a common shock to several economies.\(^1\)

Identifying and explaining contagion has several important theoretical implications for

Sovereign Bond Spreads: 1995-2000

Source: JP Morgan.

Figure 1: Sovereign Bond Spreads

the area of international finance. First, as discussed in Valdes (1996), contagion modifies the credit constraints faced by emerging economies. In general credit constraints originate when problems of sovereign risk modify the formulation of emerging economies’ dynamic optimal plans. When contagion is present, these constraints might vary through time for reasons beyond domestic fundamentals of a particular economy. The economic analysis of developing economies need to consider this possibility, and its impact on the formulation of their optimal dynamic plans. Second, as Goldstein and Pauzner (2001) argue, the existence of contagion modifies the extent to which diversification of risk is possible for investors. In the presence of contagion, the globalization of capital markets bears a cost due to the positive correlation between assets that would be otherwise independent. Contagion reduces the means to diversify risk. This reduction introduces questions about the optimal degree of openness of financial markets. Third, identifying the mechanisms that lead to contagion allows for more precise policy evaluations faced by international organizations. For example, if contagion is explained only by shifts in market sentiments, then information disclosure standards might be justified. On the other hand, if contagion is explained by financial links, then the imposition of capital controls might be desirable. Additionally under the financial
links explanation, country bailouts might be justified in order to reduce the impact of the
crisis over the international investors’ community and avoid crises of systemic proportions.
Finally, if contagion is explained by trade links, bailouts would not be the best course of
action. In this latter case, policies directed to enhance trade diversification would be more
effective.

The simultaneity of crises has mainly affected emerging countries as in the cases of the
Latin American Debt crisis in 1982, the Mexican Crisis in 1994-1995, the East Asian Crises
in 1997, and the Russian crisis in 1998\textsuperscript{2}. However, in the case of the Russian crisis, even
developed countries were affected: The crisis spread to Brazil in 1999 and hit the US as a
correction in asset prices.

These financial crises have inspired a large economic literature addressing the issue of
contagion. However, despite the huge attention that the subject has received, the ori-
gin, severity and extent of contagion of crises across economies has not been \textit{endogenously}
explained by any model. Instead, most models analyze contagion in the framework of a
multiple equilibria world as the result of an exogenous change in investors’ perceptions.
The disadvantage of this approach is that in most cases, this framework is equally consis-
tent with the occurrence of contagion and the non-occurrence of contagion. A few models,
on the other hand, have attempted a general equilibrium approach to understand the role
of links between countries in explaining contagion. But to generate contagion, these models
impose an exogenous correlation between emerging economies’ asset returns. In contrast,
in the present work, emerging economies’ returns are determined endogenously within the
model.

Most of the few general equilibrium models to date that have considered the role of
the fundamental economic links across countries have focused on explaining contagion as a
consequence of the transmission of crises through commercial links across economies, and
are silent about the role of financial links. Moreover, empirically trade links cannot account
for the severity and extent of contagion across economies that do not have strong trade
linkages with one another. In most recent cases of contagion, trade links across infected
countries were small or nonexistent. Even in the case where trade links were economically
relevant, the links alone were not strong enough to account for the severity of contagion.

There are a few other shortcomings of the existing general equilibrium models: These

\textsuperscript{2}The Mexican crisis in 1994 had was its know as the Tequila effect, that hit the economies of Argentina
and Brazil. The Russian crisis spread to countries in Latin America, and Europe.
models abstract from uncertainty and financial market imperfections\textsuperscript{3}. However in the RBC literature the inclusion of uncertainty and financial market imperfections has proved necessary to match the stylized facts of the business cycle. Also, the models that consider financial links focus on investors—why investors act in a way that causes contagion, and how contagion affects those investors. However, these models do not take into account the effects of contagion on emerging economies’ optimal plans.

In contrast, the model developed in the current paper emphasizes financial market links across countries in a general equilibrium setting where the stochastic processes of the emerging economies’ assets are endogenously determined. In addition, the model in this paper undertakes a quantitative analysis of the impact of contagion on emerging economies. The model considers how contagion can explain co-movements in the price of emerging economy bonds, capital flows, output and consumption—beyond the level explained by a country’s own fundamentals.

Within the present model, the framework is one of a set of small open economies with stochastic endowments. These small open economies have access to an international credit market populated by international lenders. International lenders are assumed to be risk averse, with preferences that exhibit a decreasing absolute risk aversion in wealth. Due to the fact that the number of countries is finite, international lenders are not able to completely diversify the risk of their investment in any country\textsuperscript{4}. There is a problem of enforcement in the sense that international lenders cannot force the small open economies to repay their debts. These economies repay because it is in their interest to do so: If any economy defaults it is temporarily excluded from the world asset market. Countries weight the benefits and costs of default, and decide to repay or not. This context forces international lenders to consider the risk of default when choosing their portfolio. Any type of reallocation of the international lenders portfolio has effects over several countries at the same time, therefore the risk of default is endogenously determined by the own economy fundamentals, and by the fundamentals of other economies because those fundamentals determine the risk of default of those countries, which might modify the portfolio choice.

\textsuperscript{3}Only Paasche (2001) has considered financial market imperfections. In Paasche, endogenous credit constraints act as a propagation mechanism for contagion caused by trade links. His model, like all others of this type, is silent about the existence and importance of financial links across economies.

\textsuperscript{4}As argued by Eaton and Gerzovits (1981), if international lenders were to be risk neutral agents, this characteristic would be equivalent to the statement that the risk of an individual default is uncorrelated with market risk, allowing a perfect diversification of default risk. This assumption is clearly at odds with the wide empirical evidence (which will be discussed later in the paper) that the occurrence of crisis in one country (which affects the risk of the investment in that country) increases the probability of crisis in other countries.
of the international lenders, and in that way the availability of financial resources to any emerging economy.

Within this framework, income shocks to an emerging economy generate changes in the risk of default in that economy. Through financial links, these changes in turn impact other emerging economies. Financial links generate contagion through two channels:

(i) The **Wealth** channel of contagion: If a income shock in the first country generate losses for international lenders, for example when the shock forces the first country into default, then depending on the preferences of the lenders, the negative wealth effect of the shock might reduce lenders’ tolerance for risk. A reduction in tolerance for risk would make lenders shift away from risky investments (countries) toward riskless (T-Bills). Countries that did not default or face an income shock would face a reduction in the amount of resources available to borrow from, and contagion occurs.

(ii) The **Portfolio Recomposition** channel of contagion: The risk of default might be correlated across countries because income process are correlated, or solely because the possibility of contagion caused by financial links across countries (as the previously explained wealth channel, or trade links between the economies). In the case where default risk are positively correlated, an increase in the risk of default in the first country generates, for a given level of wealth of the lenders, a increase in the overall risk of their portfolio as lenders expect other countries risk of default to also increase. In this case, the change in the risk of default in one country modifies the optimal portfolio of international lenders. As lenders adjust their portfolios, countries which did not face an income shock nonetheless face a reduction in the amount of resources available to borrow from, and contagion occurs.

The model in this paper extends the model of Eaton and Gersovitz (1981) in order to consider sovereign bond markets in a multi-country framework. This type of model allows for the case of open small economies an endogenous determination of the price of one period non-contingent discount bonds as a function of the economy’s default risk. Through the consideration of financial links across economies, the default risk of any economy becomes a function not only of the domestic fundamentals but also a function of investors characteristics, and the fundamentals of countries which share lenders with the domestic country.

The main mechanism through which financial contagion occurs, the wealth channel, is analyzed in the papers of Goldstein and Pauzner (2001), Kyle and Xiong (2001), and La-
gunoff and Schreft (2001), these papers show that if investors’ preferences exhibit decreasing absolute risk aversion the optimal response of the investors to financial losses is to reduce their risky investments, as consequence of the reduction on their tolerance toward risk at lower levels of wealth.

The secondary financial channel of contagion considered in this paper, the portfolio recomposition channel, is studied in the theoretical papers of Choueri (1999), Schinasi and Smith (1999) and Kodres and Pritsker (2002). Using a static partial equilibrium approach where the determination of asset returns is exogenous to the model, these papers highlight the fact that contagion might be successfully explained by standard portfolio theory: in order to reestablish the optimal degree of risk exposure in their portfolio after a negative shock to the return of the assets of some economy, it is optimal for investors to liquidate their holdings of assets with expected returns that exhibit some correlation with the expected return of the crisis country—whether this correlation is a consequence of contagion or just correlated fundamentals.


Kaminsky and Reinhart (1998) find that the probability of crisis for any country is the highest when another country that shares investors/lenders is in crisis. No other shared characteristic with the crisis country (e.g. region, macroeconomic practices, trade links, etc.) has as large an impact on the probability of crisis. Van Rijckeghem and Weder (1999) present evidence that for the Mexican, Asian and Russian crises spill-overs through common bank lending were more significant in explaining contagion than trade linkages and macroeconomic similarities. Kaminsky, Lyons and Schmukler (1999) find evidence that individual share holders of open-end mutual funds followed contagion strategies in the case of the Russian crisis, and to lesser degree in the Mexican crisis: individual investors in mutual funds (not mutual fund managers) sold securities from several emerging markets when a crisis affects one of the countries. In the aftermath of the Mexican, Asian, and Russian crises, Kaminsky Lyons and Schmukler (2000) find that when open-end mutual funds adjusted their portfolios they considered not only the degree of fragility in fundamentals of the economies, but also factors emphasized by financial channels of contagion—openness and liquidity of the markets, as well as the level of country risk of the economies. Kaminsky and Reinhart (2000) find that during the Russian-LCTM crises reductions in the risk exposure of investors portfolios drastically reduced the liquidity of international capital markets and increased
their volatility. Hernandez and Valdes (2001) find the presence of common lenders seems to explain almost all contagion episodes during the Asian, the Russian, and the Brazilian crises. Common lenders are also highly significant in explaining contagion in stock markets.

The results of this paper are consistent with the empirical evidence regarding contagion as consequence of financial links.

First, since investors’ preferences exhibit decreasing absolute risk aversion, they are able to tolerate more default risk when they are wealthier. So there is a positive correlation between lenders’ wealth and their investment in emerging economies. Therefore both capital flows to emerging economies and the equilibrium price of sovereign bonds are increasing functions of investors’ wealth levels. This result implies that the current model can explain the high correlation of sovereign bond spreads and capital flows across emerging economies. Furthermore, the high correlation between investors’ wealth and emerging economies financing conditions can account for the simultaneity of crises because a default by any economy is equivalent to a negative wealth shock to the investors. This shock is transmitted to other countries via the wealth channel of contagion.

Second, because the endogenous credit limits faced by the emerging economy are a function of investors’ risk aversion, then when the probability of default increases for one country, other countries’ financing conditions may deteriorate. When the probability of default for one country increases, two opposing forces affect the financing situation of other emerging economies:

On the one hand, a decrease in the price of the sovereign bonds of the ground-zero country constitutes an expected future negative wealth shock to the investors due to the higher associated probability that this country will default, increasing the incentives of the other economies to default, and their default risk. On the other hand, an increase in default probabilities induces a substitution away from the assets of the economy whose risk increased toward the assets of the other economies. This effect would tend to increase the set of financial contracts available to (other) emerging economies. If the first effect dominates contagion is observed: the correlation of capital flows across emerging economies is

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5This result is consistent with empirical findings which demonstrate a positive relation between proxies of investors wealth (like developed economies’ GDP or stock indexes) and capital flows to emerging economies. See for example Goldberg (2001), Hernandez, Mellado and Valdes (2001), Calvo, Fernandez-Arias, Reinhart, Talvi (2001), Kang, Kim, Kim and Wang (2003), FitzGerald, and Krolzig (April 2003), and Mody and Taylor (2004).

This result is also consistent with the empirical literature on the determination of sovereign credit spreads for emerging economies. See, for example, Warther (1995), Ferruci, Herzberg, Soussa, and Taylor (2004), FitzGerald, and Krolzig (April 2003), and Westphalen (2001).
positive. If the second effect dominates, "flight to quality" is observed: emerging economies with robust fundamentals receive capital flows when several other countries are affected by financial crises. In the numerical simulations in the present paper contagion dominates the flight to quality. In this case, the transmission of crisis is consistent with the recomposition channel of contagion.

Third, the likelihood of observing default in equilibrium for any emerging economy is a function not only of investors’ characteristics and the economy’s own fundamentals, but also of other emerging economies’ fundamentals. In the numerical simulations in the present paper, default is more likely to be an equilibrium outcome when the fundamentals of other economies deteriorate.

The paper proceeds as follows: Section II documents some episodes of simultaneous crises across economies and discusses some empirical results which motivate the study of contagion; section III develops the model; section IV characterizes the equilibrium of the model; section V presents the numerical results of the paper; and section VI concludes.

2 Evidence Of Contagion: Time Clustered Crisis

International capital markets are highly volatile. A striking feature of this volatility is that most episodes of crisis within the last two decades have not been restricted to individual countries, or even regions. This time-clustering of crises has led economists to borrow from epidemiology the term contagion. Some examples of time clustered crises follow.

The Debt Crisis of 1982: Between 1979-1981 capital flows in the form of bank lending to Latin American countries reached about 6% of the region’s GDP (their peak was $41 billion in 1981). In 1982, after a tremendous hike in international interest rates, Mexico declared a moratorium on its debt, and emerging markets around the world were excluded from voluntary capital markets and forced to run current account surpluses to pay their foreign debts. The crisis affected all countries in Latin America and spread to countries as far as Nigeria, the Philippines and Yugoslavia.

The Mexican Crisis in 1994: During the period 1992-1994, Mexico’s current account deficit averaged more than 7% of the GDP. In 1995, after the crisis the country’s current account was forced into balance, and the economy experienced a negative growth of 6.5%. During the two quarters following Mexican devaluation, international mutual funds reduced their average exposure to Brazil by approximately 5%. In Argentina, the central bank lost about a third of its liquid international reserves, and the banking system lost 18% of its
deposits. Argentina’s real GDP fell by almost 5% during 1995.

The Asian crises in 1997: During the mid-1990s flows into Indonesia, Malaysia, Korea, the Philippines and Thailand averaged more than US$40 billion per annum, with a maximum of US$70 billion in 1996. During the crisis period more than US$100 billion in short term debt bank loans were recalled from these same countries.

The Russian Default/Devaluation in August of 1998: By mid-August of 1998 a severe crisis began in Russia due to fiscal imbalances, the deterioration of the capital account, the fall in international prices of Russian exports, and huge losses of international reserves. This crisis spread to Argentina, Mexico, Venezuela, Brazil, Pakistan and South Africa. As noted in Kaminsky, Lyons and Schmukler (2000), following the Russian crisis total capital inflows for Latin America diminished 35%, and short term portfolio flows—bonds, equity and bank lending—fell by 60%. Dornbush, Park and Claessens (2000) argue that the sharp reversal in capital flows to emerging economies after this crisis triggered recessions in many developing countries, and that in 1999 two fifths of the world economy experienced recession, with most GDP declines concentrated in the developing world.

As it was discuss in the introduction, the occurrence of simultaneous crises can be attributed to one of three type of reasons: i) Monsoonal effects which are due to a common shock to several economies; ii) Transmission of the crises through international fundamental economic links; iii) Changes in international investors’ perception about the outlook of an economy after a negative shock in some other economy.

Once the phenomena of time-cluster crises is observed empirical tests are necessary to determined which of the previous reasons can explain the occurrence of simultaneous crises. The empirical literature in this subject is quite large, and evidence of contagion in sovereign bonds markets is considerable: according to Valdes (1996) during the period 1986-1994 there was increased co-movement between emerging economies’ sovereign bond markets in periods of crisis; Baig and Goldfajn (1998) find that during the Asian crisis there was an increase in cross-country correlation among sovereign bond markets of the East Asian Economies affected by the crises; Edwards (1998) finds evidence of significant propagation of volatility from Mexico to Argentina bond markets during the Mexican Crisis; and Baig and Goldfajn (2000) find that during the Russian crisis there was contagion from Russia to Brazil Brady market.
3 The Model

The model is a discrete time, infinite horizon model. There are two types of agents in the model, \( J < \infty \) representative agent small open economies, and a representative risk averse international investor. In each period, each of the emerging economies receives a stochastic endowment of tradable goods. The representative agents of these economies may smooth their consumption across periods by trading non-contingent discount bonds with the representative investor but the emerging economies are not able to trade financial assets between them. For her part, the representative investor may trade assets with the emerging countries or with industrialized countries. Thus the investor must choose an optimal allocation of her portfolio between the bonds of the emerging economies and bonds of the industrialized countries, denominated hereafter as T-Bills. By having only a finite number of countries the possibilities of diversification for the investors are limited. The consideration of a finite number of countries can be justified by the belief that there are fixed costs in gathering information about the emerging economies. The existence of these fixed costs of gathering information prevents international lenders from holding a fully diversified portfolio.

The market for T-bills, \( \theta^{TB} \), will not be modeled explicitly. Debt contracts between the representative investor and industrialized countries are assumed to be enforceable, and the representative investor is a price taker in the market for T-Bills. The price of T-Bills, \( q^I \), which is not determined endogenously in this context, is assumed to be deterministic. Therefore T-Bills are riskless assets.

Bonds of emerging economies, \( b_j \), on the other hand, are risky assets because debt contracts between the representative investor and the emerging economies are not enforceable. As a consequence, there is a one sided commitment problem. While the representative investor is able to commit to honor her debt obligations with the emerging economies, the representative agents of the emerging countries are not able to commit to honor their obligations with international investors. Therefore in each period, the representative agent of each emerging economy compares the costs and benefits derived from the repayment of her obligations. The decision between repayment or default is made individually by each agent of each emerging economy. Each agent of any economy makes her decision, taking as given the decision of the other agents in her economy. However given that all agents in a given economy are identical and do not follow mixed strategies, it is possible to focus attention on the problem of the representative agent of each economy.

If any economy defaults, international investors are able to collude to punish her. As a
consequence of default, it is assumed that investors will collude to exclude the defaulting country forever from the financial markets. Since all investors behave in the same exact way, it is possible to focus on the representative international investor.

Both, the representative lender, and the representative agents of the economies take as given the price function of each emerging economy’s non-contingent discount bonds, $q_j^6$.

As laid out here, the asset market is imperfect in three different ways. First, there is a one-sided commitment problem which implies that debt contracts with emerging economies are not enforceable. Second, markets are incomplete because the only traded assets are one period no-contingent bonds, and risk free T-Bills. Therefore the representative investor is not able to insure away the income uncertainty specific to the emerging countries. Third, the market structure of the financial market is non-competitive: investors form a cartel that colludes to punish any deviant investor or borrower.

In this model the state of the world is complex object defined as follows:

**Definition 1** The state of the world, $S = (s, W, \psi)$, is given by the realization of the emerging economies’ fundamentals, $s = s_1 \times s_2 \times \ldots \times s_J$, the representative investor’s asset position or wealth, $W$, and a probability measure $\psi$ on $s \times W \times \Gamma$ where $\Gamma$ is a borel $\sigma$-algebra. In this model $s_j = (b_j, y_j, d_j)$ where $b_j$ is economy’s $j$ asset position, $y_j$ is economy’s $j$ endowment, and $d_j$ is a variable that describes if the economy $j$ is in default or repayment state.

### 3.1 International Lenders

There are a large but finite number of price-taking identical investors. Investors collude in order to punish any borrower that defaults in her debts or any investor that lends to a borrower who has previously defaulted, so that a defaulting country is temporarily excluded from the financial markets.\footnote{As in the papers on sovereign debt literature of Kletzer and Wright (2000) and Wright(2002), no investor will deviate from this punishment as long as deviations are punished by the remaining investors. Punishment is achieved by inducing the representative agent of the emerging economy to default on her debts with the deviant investor. The colluding investors induce the emerging economy by offering her a new financial contract with slightly better terms. In this case investors will never deviate. The assumption of temporarily exclusion is supported by the data: Empirical evidence suggests that once a country defaults, that country is excluded from the credit market for an average of 5.4 years (Gelos, Sahay, and Sandleris 2003).}

\footnote{Because of the enforcement problems the price of the bonds of any emerging economy depends on the amount of borrowing of that economy.}
The representative investor is a risk averse agent whose preferences over consumption are defined by a constant relative risk aversion (CRRA) periodic utility function with parameter $\gamma^L > 0$. The investor has perfect information regarding the income process of the emerging economy, and in each period the investor is able to observe the realizations of this endowment.

The representative investor maximizes her discounted expected lifetime utility from consumption

$$\max_{c_t^L} E \left[ \sum_{t=0}^{\infty} \beta_t^L v \left( c_t^L \right) \right]$$

where $c_t^L$ is the investor’s consumption. The period utility of this agent is given by $v(c_t^L) = \left( \frac{c_t^L}{1 - \gamma^L} \right)^{1 - \gamma}``. The representative investor is endowed with some initial wealth $W_0$, at time 0, and in each period, the investor receives an exogenous income $X$.

Because the representative investor is able to commit to honor her debt, she can borrow or lend from industrialized countries (which are not explicitly modeled here) by buying T-Bills at the deterministic risk free world price of $q^f$. The representative investor can also invest in non-contingent bonds of the emerging economy. These bonds have an endogenously determined stochastic price of $q_j$. In each period the representative investor faces the budget constraint

$$W + X = c_t^L + \sum_{j=1}^{J} d_j q_j \theta_j^T + q^f \theta_T^{TB}$$

where $W$ is investors wealth at time $t$, $\theta_j^T$ is the portfolio allocation to the emerging country and $\theta_T^{TB}$ is the investor’s allocation to the riskless asset. $d_j$ is a variable that determines the default/repayment state of the emerging economy $j$ in the current period. $d_j$ is an indicator function that represents the emerging economy’s $j$ repayment/default decision in the current period. $d_j$ takes the value of 1 when the small open economy $j$ chooses to repay its debts, and takes a value of 0 otherwise.

It is assumed that investors cannot go short in their investments with emerging economies. Therefore whenever the emerging economy is saving, the representative international investor receives these savings and invests them completely in riskless bonds (T-Bills). The representative investor does not use these resources to go long in T-Bills. This assumption implies that $\theta_j^T \geq 0$ for all $t$ and $j$.8

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8This assumption does not seem to be inconsistent with reality. For example, while mutual funds are strictly restricted by The Investment Company Act in their ability to leverage or borrow against the value of securities in their portfolio, hedge funds and other types of investments face no such restrictions. Since
The law of motion of the representative investor’s wealth is given by

\[ W' = \sum_{j=1}^{J} d_j \theta_j' + \theta^T B. \]  

(3)

The optimization problem that the representative investor faces can be described as one in which in each period \( t \) the representative international investor optimally chooses her portfolio according to her preferences in order to maximize her discounted expected lifetime utility from consumption, subject to her budget constraint, the law of motion of her wealth, and given \( W_0 \). This dynamic problem can be represented recursively by the Bellman Equation

\[ V^L(S) = \max_{\{\theta_j'\}_{j=1}^{J}, \theta^T B} \left\{ \theta_j' \sum_{j=1}^{J} d_j \theta_j' \theta^T B \right\} v(c^L) + E\beta V^L(S') \]  

(4)

The stochastic dynamic problem for the representative investor is characterized by the first order conditions for this optimization problem:

- For \( \theta^T B \)
  
  \[ q^I v_c L (c^L) = \beta L E \left[ v_c L (c^L') \right] \]  

(5)

- For \( \theta_j' \)
  
  \[ d_j \left[ - q_j v_c L (c^L) + \beta L E \left[ v_c L (c^L') d_j' \right] \right] = 0 \]  

(6)

According to (5), the investor chooses an allocation to the riskless asset such that the discounted expected marginal benefit of future consumption equals the marginal cost of current consumption. The set of \( J \) equations (6) determine the allocation of the resources of the representative investor to each one of the \( J \) emerging countries. Unless emerging country \( j \) is not in a default state, i.e. \( d_j = 1 \), emerging country \( j \) does not belong in the investment set of the international investors. If country \( j \) has not defaulted, then the \( j \)th equation (6) also equates the marginal cost of allocating wealth to bonds issued emerging country \( j \) to the discounted expected marginal benefit of this investment. The benefit of international investments like hedge funds are not subject to these type of regulations, it seems reasonable to have the simplifying assumption that international investors are able to leverage the riskless asset, \( \theta^T B \), but must have a non-negative position on the emerging economy’s asset.
this investment is realized only in those periods in which the emerging economy $j$ optimally chooses to repay its debts ($d_j' = 1$).

For the case in which $d_j = 1$, the $j^{th}$ equation (6) highlights the fact that the endogenous risk of default by the emerging economy $j$ — i.e. the case for which $d_j' = 0$ for some state of the world in the next period—will reduce the representative investor’s expected marginal benefit of investing in the emerging economy. Everything else equal, this result will tend to reduce the allocation of resources to emerging economy $j$ relative to the case where this emerging economy could commit to repayment.

Define $Ed_j' = 1 - \delta_j$ where $\delta_j$ is the probability that emerging economy $j$ will default in the next period.

For the case in which $d_j = 1$, the $j^{th}$ equation (6) highlights the fact that the endogenous risk of default by the emerging economy $j$ — i.e. the case for which $d_j' = 0$ for some state of the world in the next period—will reduce the representative investor’s expected marginal benefit of investing in the emerging economy. Everything else equal, this result will tend to reduce the allocation of resources to emerging economy $j$ relative to the case where this emerging economy could commit to repayment.

Define $q_j^{RN}$ as the equilibrium price of the emerging economy’s bonds that would prevail in a world with risk neutral lenders. For a risk neutral investor, the present value of one unit of a bond issued by a emerging economy that cannot commit to repay is given by

$$-q_j^{RN} + q^f Ed_j'$$

where $q^f$ is used as the discount factor. This factor represents the opportunity cost for the representative investor of her investment in emerging economy $j$'s bonds. Given the assumption that the investor is a price taker, and that the price function of the bonds of any economy is the equilibrium result of a competitive credit market a risk neutral representative investor would make zero profits. Therefore (7) implies

$$q_j^{RN} = q^f (1 - \delta_j)$$

which establishes that for the case of a representative risk neutral investor, the price of a discounted one period non-contingent bond of any economy is equal to its opportunity cost.

It is possible to manipulate the $j^{th}$ equation (6) to get

$$q_j = \frac{\beta_L \text{Cov} \left[ v_{c \ L} \left( c \ L' \right) d_j' \right]}{v_{c \ L} \left( c \ L' \right)} + q^f (1 - \delta_j) = \frac{\beta_L \text{Cov} \left[ v_{c \ L} \left( c \ L' \right) d_j' \right]}{v_{c \ L} \left( c \ L' \right)} + q_j^{RN}$$

It is clear from equation (8) that unless the probability of default is positive, the price of the emerging economy’s bonds is equal to the price of the bonds of industrialized countries. For the simpler case of one sovereign emerging economy that interacts with risk averse international lenders, as in Lizarazo(2005) as consequence of the lenders’ risk aversion, the
price of sovereign bonds of any emerging economy, $q_j \left( \delta_j \left( S, b'_j \right) \right)$, is always lower or at best equal to price of those same bonds traded with a representative risk neutral investor, $q_j^{RN} \left( \delta_j^{RN} \left( S, b'_j \right) \right)$. This latter implication holds true because $\text{Cov} \left[ v_c, L \left| d'_j \right. \right] \leq 0$.9

From the set of equations (8) is possible to see that bond prices of any emerging economy are a function not only of investors’ risk aversion and the economy’s own fundamentals.

Because consumption of the investor is also a function of their wealth and of their investments in other economies, sovereign bond prices of the emerging economy $j$ are also a function of those variables.10

Because both, investors’ wealth, and fundamentals of other emerging economies in the investors’ portfolio have an effect on the determination of bond prices of some economy $j$, it is clear that sovereign bond prices across economies that share investors must be correlated.

The discussion on the way in which other countries fundamentals affect the determination of economy $j$’s bond prices, and debt flows would be postponed until the section on the characterization of contagion channels.

---

9When the emerging economy does not find it optimal to default next period in any state of the world, then $d'_j = 1$ for all states. Therefore $\text{Cov} \left[ v_c, L \left| d'_j \right. \right] = 0$. On the other hand, when next period exist states of the world in which the emerging economy would optimally choose to default, then for the states in which it is not optimal to default, $d'_j = 1$. In this case, next period wealth of the representative investor is given by

$W' \mid d'_j = 1 = \theta' + \theta T_{Br}$

and next period wealth of the representative investor is given by

$W' \mid d'_j = 0 = \theta T_{Br}$

It is obvious that

$W' \mid d'_j = 1 > W' \mid d'_j = 0$

Therefore it must hold that

$\left[ c^L \mid d'_j = 1 \right] \geq \left[ c^L \mid d'_j = 0 \right]$ implying

$\left[ v_c, L \mid c^L \mid d'_j = 1 \right] \leq \left[ v_c, L \mid c^L \mid d'_j = 0 \right]$

As a consequence, for higher $d'_j$, we have lower $v_c, L \mid c^L$. Clearly for this case $\text{Cov} \left[ v_c, L \mid c^L \mid d'_j \right] < 0$.

10The way in which sovereign bond prices depend on investors’ wealth is discussed in depth on Lizarazo (2005). Whenever investors are wealthier, the marginal cost for them in terms of utility of an additional unit of investment on sovereign bonds is relatively low. Taking as given the price of the bonds of an emerging economy $j$, it must hold that the net flow of resources to the economy i.e., $-q_j b'_j$ is increasing in investors wealth. As it would be evident once the emerging economies’ problem is described, the benefits for the economy from paying her debts are also increasing in investors’ wealth and therefore incentives to default decrease with investors’ wealth. In equilibrium sovereign economies bond prices are increasing on investors’ wealth.
3.2 Sovereign Countries

The representative agent of each emerging economy $j$ maximizes her discounted expected lifetime utility from consumption

$$\max_{\{c_{j,t}\}_{t=0}^{\infty}} E_{\tau} \sum_{t=0}^{\infty} \beta^t u(c_{j,t})$$  \hspace{1cm} (9)

where $0 < \beta < 1$ is the discount factor and $c_j$ is the $j$ emerging economy’s consumption at time $t$. The emerging economy’s periodic utility takes the functional form

$$u(c_j) = c_j^{1-\gamma} \frac{1}{1-\gamma}$$

where $\gamma > 0$ is the coefficient of relative risk aversion.

In each period, each economy $j$ receives a stochastic stream of consumption goods $y_j$. This endowment is non-storable, and it is independently distributed across emerging economies; realizations of the endowment are assumed to have a compact support; and the endowment follows a Markov process drawn from probability space $(y_j, Y_j(y_j))$ with a transition function $f(y'_j | y_j)$.

In each period, based on the stochastic endowment $y_j$, the economy decides how much to consume $c_j$. The economy can consume $c_j > y_j$ by trading one period non-contingent discount bonds $b'_j$ at a price $q_j$ with international investors. The economy may only trade bonds if in period $t$ the economy is not in default state.

At any point in time the emerging economies differ in their endowment and their asset position, therefore it is necessary to have a probability measure $\psi$ on $s \times W \times \Gamma$ where $\Gamma$ is a borel $\sigma$-algebra. This probability measure is known by all the agents and its law of motion of the distribution is given by:

$$\psi' = H(\psi)$$  \hspace{1cm} (10)

In equilibrium, the price of bonds is determined by both investors and the emerging economy. As a consequence of the commitment problems, the price of the emerging economy’s bond might be different depending on whether the economy is saving or borrowing. If $b'_j > 0$, the country is saving, and because the international investor is able to commit, there is no risk of default on such a bond. In this case, the emerging economy’s bond is identical to the bonds issued by industrialized markets; therefore, because the representative investor is a price taker, in equilibrium the bond price of a emerging economy with no default risk is the same as the bond price of industrialized countries. Consequently, the price of a bond with a positive face value is equal to the price of a T-Bill, so $q_j = q^f$. 

16
If $b_j' = 0$, the emerging economy is not borrowing and there is no risk of default because it is not optimal for the emerging economy to declare default on a debt of size 0. If the economy were to declare default in this circumstance, there would be no change in the present pattern of consumption, but a reduction in the opportunities of consumption smoothing in the future.

If $b_j' < 0$ the emerging country is borrowing. In this case, because emerging economies cannot bind themselves to honor their debts, the emerging country $j$ might default next period. There might be values of $b_j' < 0$, for some given state of the world, $S$, such that the representative agent of the economy never finds it optimal to default. In this case the bonds issued by the emerging economy do not involve any default risk, and therefore $q_j = q^f$. However for the same state of the world, $S$, some other values of $b_j' < 0$ might imply that the emerging economy will find it optimal to default on her debts in some states of the world next period $S'$. In this case, in order to induce international investors to buy the emerging economy’s bonds, the price of such bonds needs to be lower than the price of a T-Bill, $q_j < q^f$. Finally, for the same state of the world, $S$, there might be values of $b_j' < 0$ such that once the debt is due the economy would not choose to repay in any state of the world next period, $S'$. In this case $q_j = 0$.

Based on this logic, the price of the emerging economy’s bonds is a function not only of the state of the world, $S$, but also of $b_j'$. Therefore the agents in the credit market observe the menu of equilibrium bond prices for each level of borrowing $b_j'$ for every state of the world, $S$.

The resource constraint of the emerging economy is given by

$$c_j = y_j + d_j (b_j - q_j b_j')$$

(11)

where $d_j$, which has been defined in the investor’s section, describes the state of economy with respect to participation in international financial markets. If $d_j = 1$, the economy is not in a default state. If $d_j = 0$ the emerging economy is in default and this country is in temporarily financial autarky. Once a country defaults (even if the default is partial), that country is temporarily excluded from access to the credit market, and the country remains in a state of default for a random number of periods. In such case the country is not able to smooth its consumption, and it is limited to consume its stochastic endowment for a period of time.

Under this framework, the optimization problem of the emerging country $j$ can be represented recursively by the following Bellman equation

$$V_j(S) = \max \{V_j^C(S), V_j^D(S)\}$$

(12)
and

\[
V_j^C(S) = \max_{c_j, b_j'} \ u(c_j) + \beta E V_j(S' | S)
\]

\[
s.t. \quad c_j = y_j + b_j - q_j b_j' \quad \& \quad b_j' \geq b_j
\]

(13)

where \( V_j^C(S) \) is the value for economy \( j \) of not defaulting and \( V_j^D(S) \) is the value of defaulting in the current period.

**Definition 2** The value for the emerging economy \( j \) of default is given by

\[
V_j^D(S) = u(y_j) + \beta E[\theta V_j^C(S' | S) + (1 - \theta) V_j^D(S' | S)].
\]

For the emerging country the decision of default/repayment depends on the comparison between the continuation value of the credit contract, \( V^C(S) \), versus the value of opting for financial autarky \( V_j^D(S) \). The decision of current default/repayment takes the functional form:

\[
d_j = \begin{cases} 
1 & \text{if } V_j^C(S) > V_j^D(S) \\
0 & \text{otherwise}
\end{cases}
\]

(14)

The set of equations (13) corresponds to the “natural” debt limit discussed in Ayagari (1993), which prevents the representative agent of the emerging economy \( j \) from running ponzi games. In the current model, this constraint would not be binding. Instead a tighter credit limit is determined endogenously in the model.

The stochastic dynamic problem for the emerging economy \( j \) is characterized by the Euler equation (conditional on not defaulting in the current period):

\[
u_c(c_j(S))q_j(S; b_j') (1 + \frac{\partial q_j(S; b_j')}{\partial b_j'} \frac{b_j'(S)}{q_j(S; b_j')}) = \beta E[u_c( c_j'(S') d_j'(S'))]
\]

(15)

and equations (11) and (14).

The Euler equation (15) equates the marginal benefit of one unit of current consumption to the discounted expected marginal cost of giving up one unit of future consumption. Because of the commitment problem, this cost is experienced only in those states in which the emerging economy \( j \) optimally chooses to repay its debt, i.e. only on those states in which \( d_j' = 1 \).

The Euler equation (15) highlights some important features of the model. First, the decision not to default in the current period does not imply that the economy will not
default in the future (i.e. \( d_j \) might be 0 for some states of the world). Second, for an economy which has the possibility of default, the optimal path of consumption is different from the optimal path of consumption of an otherwise identical economy which cannot default. To see this difference, consider the following. A small economy that can commit to repay its debts takes the price of the bonds that issues as given, and it is able to borrow or lend always at that price. The Euler equation for such an economy is simply given by

\[
q^f u_c(c_j) = \beta E u_c(c_{j}').
\]

Comparison of equations (15) and (15) shows that there are two additional effects which modify the inter-temporal savings decision for an economy which cannot commit to repayment. First, the cost of borrowing in the current period is only experienced in states of the world for which future repayment is optimal. All else equal, this effect suggests that an economy that cannot commit to repayment will tend to borrow more than an economy that can commit. Second, for an economy that cannot commit, the price of the bonds depends on the borrowing decisions of that economy. This result can be seen in equation (6). From equation (6), it is clear that the borrower’s limited liability reduces the investor’s incentive to invest in the emerging economy.

Equations (15) and (6) make clear that for the case of an economy that cannot commit to repayment, when there exist levels of \( b_j \) in which the emerging economy finds it optimal to default in some states of the world, then the price of bonds depends not only on the emerging economy’s fundamentals, but on the representative investor’s consumption (that depends on the investors level of wealth and her portfolio allocation in other emerging economies) and the investor’s risk aversion. This case is very different from the case of a small open economy that can commit. In the latter environment, the assets of the emerging economy are riskless from the point of view of the representative investor. Therefore, as long as the representative investor is a price taker, the price of emerging economy’s bonds is equal to the price of the industrialized countries’ bonds. And bond prices are independent of the investors’ risk aversion, wealth and portfolio allocation.

Another feature of this model is that the emerging economy only defaults when it is facing capital outflows. In this case, \( d_j(s) = 0 \) implies that for all the financial contracts available to the economy \( b_j - q_j(S; b_j'(S))b_j'(S) < 0 \). Intuitively, whenever the emerging economy decides to default, the value of default must be at least as good as the value of the optimal financial contract available to this country \( V_j^C(S) \leq V_j^D(S) \). However if any available financial contract allows for capital inflows to the emerging economy, then by choosing that contract the economy not only can consume more in the current period than under the default state \( (c_j > y_j) \), but in the next period the economy is guaranteed at
least the same level of satisfaction as under default (because the economy has the option of defaulting in the next period). Therefore for any state of the world \((S)\), whenever there are financial contracts \(\{q_j(S; b'_j(S)), b'_j(S)\}\) such that \(b_j - q_j(S; b'_j(S))b'_j(S) > 0\), default is not an optimal decision.

4 Equilibrium

The recursive equilibrium in this model is given by prices \(q^f\), and \(\{q_j\}_{j=1}^J\), quantities \(\{c_j\}_{j=1}^J; \{b'_j\}_{j=1}^J; \{\theta'_j\}_{j=1}^J; \theta'^TB'\), the law of motion of the distribution \(H(\psi)\), an aggregate law of motion of the world economy \(Q(S, \Gamma; \{b'_j\}_{j=1}^J, W')\), and an updating operator \(T(\psi, Q(S\Gamma; \{b'_j\}_{j=1}^J, W'))\) such that:

\[
H(\psi) = T(\psi, Q(S, \Gamma; \{b'_j\}_{j=1}^J, W')).
\]

which solve the emerging economies’s problem and the representative investor’s problem.

The equilibrium prices and quantities must also clear asset markets:

\[
\begin{align*}
b'_j &= -\theta'_j & \text{if } b'_j < 0, \quad j = 1, 2, \ldots J. \\
0 &= -\theta'_j & \text{if } b'_j \geq 0, \quad j = 1, 2, \ldots J.
\end{align*}
\]

Equations (16) and (16) imply that in equilibrium each emerging economy \(j\) and the representative investor agree on a financial contract, \(b'_j\) and \(q_j\), that is optimal for both agents.

Definition 3 For a given level of wealth, \(W\), and the fundamentals of other emerging economies in the investor’s portfolio, the default set \(D_j(b | W, \psi, \{s_k\}_{k=1, k\neq j})\) consists of the equilibrium set of \(y_j\) for which default is optimal when emerging economy \(j\)’s asset holdings are \(b_j\):

\[
D_j(b_j | \{s_k\}_{k=1, k\neq j}, W, \psi) = \{y_j \in Y_j : V^C_j(S) \leq V^D_j(S)\}.
\]

Equilibrium default sets, \(D_j(b'_j | \{s'_k(S)\}_{k=1, k\neq j}, W'(S), \psi'(\psi))\), are related to equilibrium default probabilities, \(\delta_j(S' | S)\), by the equation

\[
\delta_j(S' | S) = El'_j(S' | S) = \int f(y'_j | y_j)dy'_j.
\]

20
If the default set is empty for $b'_j$, then for all realizations of the economy $j$’s endowment $d'_j = 1$ and the equilibrium default probability $\delta_j(S' | S)$ is equal to 0. In this case, it is not optimal for the economy to default in the next period for any realization of its endowment, $\text{Cov} \left[ v_c L \left( c^{L'} \right) d'_j \right] = 0$ and $q_j = q^f$. On the other hand, if the default set includes the entire support for the endowment realizations, i.e. $D_j(\{b'_j \mid \{s'_k(S)\}_{k=1,k\neq j}, W'(S), \psi'(\psi)\} = Y_j$, then $d'_j = 0$ for all realizations of the economy’s endowment. As a consequence, the equilibrium default probability $\delta_j(S' | S)$ is equal to 1, and $\text{Cov} \left[ v_c L \left( c^{L'} \right) d'_j \right] = 0$, so $q_j = 0$.

Otherwise when the default set is not empty but does not include the whole support for the endowment realizations $0 < \delta_j(S' | S) < 1$. In this case, which was analyzed in the previous section describing the investors optimization problem, $\text{Cov} \left[ v_c L \left( c^{L'} \right) d'_j \right] > 0$, so $q_j < q^f$.

4.1 Characterization of Default Sets

The characterization of default sets is the characterization of incentives to default and therefore the characterization of endogenous default risk. In this model default risk is a function of the emerging economy’s fundamentals (the economy’s endowment process and its asset position), the characteristics of the international investor (the investor’s risk aversion and wealth), and the fundamentals of countries, $-j$, that share investors with country $j$. This section focus on the roles that investors characteristics and the own emerging economy fundamentals have in the determination of default incentives for economy $j$. The role of other economies fundamentals in the characterization of default sets will be analyzed in the section on contagion.

Endogenous Credit Constraints and Maximum Safe Level of Debt  In order to continue with the characterization of the default sets it is necessary to define two concepts, the endogenous credit constraint and the maximum safe level of debt. The endogenous credit constraint is the maximum level of assets, $b_j(\{s_k\}_{k=1,k\neq j}, W, \psi)$, that is low enough such that no matter what the realization of the endowment, default is the optimal choice and $D_j(b_j(\{s_k\}_{k=1,k\neq j}, W, \psi) | \{s_k\}_{k=1,k\neq j}, W, \psi) = Y_j$. In the other hand, the maximum safe level of debt is the minimum level of assets $\overline{b}_j(\{s_k\}_{k=1,k\neq j}, W, \psi)$ for which staying in the contract is the optimal choice for all realizations of the endowment. In this case, $D_j(\overline{b}_j(\{s_k\}_{k=1,k\neq j}, W, \psi) | \{s_k\}_{k=1,k\neq j}, W, \psi) = \emptyset$. Finally, because the value of the credit
contract is monotonically decreasing in $b_j$, it is obvious that

$$b_j(\{s_k\}_{k=1,k \neq j}^J, W, \psi) \leq b_j(\{s_k\}_{k=1,k \neq j}^J, W, \psi) \leq 0$$

**Proposition 1** For any state of the world, $S$, the endogenous credit constraint $b_j(\{s_k\}_{k=1,k \neq j}^J, W, \psi)$, and the maximum safe level of debt $\overline{b}_j(\{s_k\}_{k=1,k \neq j}^J, W, \psi)$ are single-valued functions.

**Proof.** To define these concepts note that the stochastic process for the endowments have a compact support. Also note that, conditional on $W$, the fundamentals of other emerging economies in the investors’ portfolio, and the distribution $\psi$, the value of the credit contract is monotonically decreasing in $b_j$. Monotonicity of the credit contract and compactness of the endowment support are sufficient conditions to guarantee that given the state of the world these critical values (i.e., endogenous credit constraint and maximum safe level of debt) are single-valued functions. □

From the previous discussion is clear that given some current level of investors’ wealth, the fundamentals of other emerging economies in the investors’ portfolio, and the distribution $\psi$ any investments in the emerging economy’s bonds in excess of $b_j(\{s_k\}_{k=1,k \neq j}^J, W, \psi)$ imply a probability of default equal to 1. These investments will have a price of 0. On the other hand, all investments in the emerging economy’s bond of an amount lower than $\overline{b}_j(\{s_k\}_{k=1,k \neq j}^J, W, \psi)$ imply a zero probability of default. These investments will have a price of $q^f$.

**Default Sets and Risk Aversion of International Investors** The degree of investors’ risk aversion is an important determinant of access of emerging economies to credit markets, and of the risk of default of the economy. In this model, the more risk averse are international investors, the higher is the default risk and the tighter is the endogenous credit constraint faced by all emerging economies.

**Proposition 2** For any state of the world, $S$, as the risk aversion of the international investor increases, the emerging economies’ incentives to default increase.

For $\gamma_L^1 < \gamma_L^2$

$$b_j(\cdot; \gamma_L^2) \geq b_j(\cdot; \gamma_L^1),$$

$$\overline{b}_j(\cdot; \gamma_L^2) \geq \overline{b}_j(\cdot; \gamma_L^1).$$
Proof. See Appendix. ■

The economic intuition behind the result is straightforward. While for the emerging economies, the value of default is not a function of the degree of risk aversion of international investors; the value of maintaining access to credit markets is decreasing in the lender’s degree of risk aversion. In order to induce a very risk averse investor to hold sovereign bonds, the representative agent of each emerging economy has to forgo much more current consumption—i.e., has to accept a very low price for her bonds. Other things equal, with lower bond prices, incentives to default are stronger. Therefore for any given state of the world, \( S \), the degree of risk in the economies is increasing in the degree of risk aversion of international investors.

As the degree of risk in the economy changes, so too will the capital flows to the economy: endogenous credit constraints \( \mathcal{B}_j \left( W, \psi, \{s_k\}_{k=1, k \neq j}; \gamma_L \right) \) for the emerging economy \( j \) are tighter the more risk averse are international investors—some contracts that are feasible under less risk adverse investors are not feasible under more risk averse investors.

The result in Proposition 2 is consistent with empirical findings which characterize the role of investor’s risk aversion in the determination of country risk and sovereign yield.\(^{11}\)

**Default Sets and Investor’s Wealth** In the present model, the investor’s wealth also affects the emerging economy’s performance. This result is formalized in Proposition 3.

**Proposition 3** Default sets are shrinking in the assets of the representative investor. For all \( W_1 < W_2 \), if default is optimal for \( b_j \) in some states \( y_j \), given \( W_2 \) then default will be optimal for \( b_j \) for the same states \( y_j \), given \( W_1 \) therefore \( D_j \left( b_j \mid W_2, \psi, \{s_k\}_{k=1, k \neq j} \right) \subseteq D_j \left( b_j \mid W_1, \psi, \{s_k\}_{k=1, k \neq j} \right) \).

For \( W_1 < W_2 \) it must hold

\[
\frac{b_j \left( W_1, \psi, \{s_k\}_{k=1, k \neq j} \right)}{\mathcal{B}_j \left( W_1, \psi, \{s_k\}_{k=1, k \neq j} \right)} \geq \frac{b_j \left( W_2, \psi, \{s_k\}_{k=1, k \neq j} \right)}{\mathcal{B}_j \left( W_2, \psi, \{s_k\}_{k=1, k \neq j} \right)} \geq \frac{\mathcal{S}_j \left( W_2, \psi, \{s_k\}_{k=1, k \neq j} \right)}{\mathcal{B}_j \left( W_2, \psi, \{s_k\}_{k=1, k \neq j} \right)}.
\]

**Proof.** See Appendix. ■

The intuition for Proposition 3 is simple: given some default risk, it is less costly (in terms of current utility) for the investor to invest in emerging economies when she is wealthy than when she is poor. So keeping constant the degree of risk that the investor faces, any investment that she is willing to undertake when she is poor she also will be willing to undertake when she is rich. Intuitively, financial contracts available to the representative agent of each emerging economy when investors are relatively rich have to be at least as good as feasible contracts when investors are relatively poor. Additionally, the previous effect implies that the emerging economies face stronger incentives to default when the wealth of the investors is relatively low. Therefore default risk is decreasing in the wealth of the investors. These two effects amplify and reinforce each other.

The previous result is a consequence of the fact that for investors the marginal cost of investing in sovereign bonds in terms of current consumption is decreasing in investors’ wealth. Given that these agents are risk averse, investing in the sovereign bonds when their wealth is low is too costly; so when the wealth of the investor falls, the resources available to the emerging economies become scarce, reducing the value for the emerging economies of participating in credit markets. In turn, because the sovereign countries have increasing incentives to default, some loans or portfolio investments that are feasible when the investor is wealthy cannot be an equilibrium outcome when the investor is poor.

Findings of several empirical papers on the literature regarding the determinants of capital flows and sovereign bonds spreads of emerging economies are consistent with the results in Proposition 3.\footnote{See, for example, Warther (1995), Westphalen (2001), Kang et al (2003), FitzGerald and Krolzig (2003), Mody and Taylor (2004), and Ferruci et al (2004).}

The results in Proposition 3 are also consistent with the evidence regarding financial contagion across countries who share investors. \footnote{See for example Kaminsky and Reinhart (1998), Van Rijckeghem and Weder (1999), Kaminsky and Reinhart (2000) and Hernandez, and Valdes (2001).}

**Default Sets and the Asset Position of the Emerging Economy** In the model, a highly indebted economy is more likely to default than an economy with lower debt. And as in models of the same type where lenders are risk neutral, default sets are shrinking in assets.

**Proposition 4** Default sets are shrinking in assets of the emerging economy. For all $b_{j,1} < b_{j,2}$, if default is optimal for $b_{j,2}$ in some states $y_j$, given $W$, and the asset position of other
emerging economies in the investors’ portfolio, then default will be optimal for \( b_{j,1} \) for the same states \( y_{j} \), given \( W \), and the asset position of other emerging economies in the investors’ portfolio. Therefore \( D_j \left( b_{j,2} \mid W, \psi, \{ s_k \}_{k=1,k \neq j}^J \right) \subseteq D_j \left( b_{j,1} \mid W, \psi, \{ s_k \}_{k=1,k \neq j}^J \right) \).

**Proof.** See Appendix. ■

This result is analogous to the result in Lizarazo (2005), and Arellano (2003), and closely related to the results in Eaton and Gersovitz (1981) and Chatterjee, et. al. (2002). The main difference in the present paper is that the result is conditioned on the level of wealth of the representative investor, and the fundamentals of other emerging economies in the investors’ portfolio. The economic intuition is as follows. While the value for the economy of fulfilling the contract is increasing in \( b_j \), the outside value of the economy is not—the value of default does not depend on \( b_j \). Therefore as the indebtedness of the economy increases, the value of the contract decreases, while the value of default remains unchanged. As a consequence, starting from an asset position \( b_j \) in which default is the optimal choice, it is clear that if the assets shrink, the value of the contract also falls. As the value of the contract falls, default will continue to be the optimal choice.

This result is consistent with the empirical literature on the determination of credit ratings and yield-bond spreads.\(^{14}\)

**Default Sets and Endowment Realization**  Default sets also depend on the realization of income. As in Arellano (2003), it is possible to show analytically that if the endowment process is i.i.d., for given \( W \), then default incentives are stronger for lower levels of income. The numerical solution of the present model extends this result to the case in which the stochastic process of the endowments follows a Markov chain with persistence.

**Proposition 5** If the endowment process is i.i.d., default incentives are stronger the lower the endowment. For all \( y_{j,1} < y_{j,2} \) if \( y_{j,2} \in D_j \left( b_j \mid W, \psi, \{ s_k \}_{k=1,k \neq j}^J \right) \) then \( y_{j,1} \in D_j \left( b_j \mid W, \psi, \{ s_k \}_{k=1,k \neq j}^J \right) \).

**Proof.** See Appendix. ■

The intuition for this result follows Arellano (2003). The main difference is that in the present context, the result is conditioned on the level of wealth of the investors, and the

\(^{14}\)See for example, Cantor and Pecker (1996), Cunningham, Dixon and Hayes (2001), Durbin and Ng (1999), and Merrick (2000)).
fundamentals of other emerging economies in the investors’ portfolio. The logic behind this results follows from the fact that default is only optimal if under all feasible financial contracts the emerging economy experiences capital outflows. In the case of a recession, capital outflows are extremely costly in terms of the welfare of a risk averse agent (because the concavity of the periodic utility); therefore at sufficiently low levels of the endowment realization, the credit market becomes a less effective tool for consumption smoothing than default.

This result is also consistent with the empirical literature on the determination of credit ratings and sovereign yields. In this literature, sovereign yield spreads increase when the economy’s fundamentals deteriorate, mainly when GDP falls.

Additionally, this result implies that because default risk is counter-cyclical, domestic interest rates are also counter-cyclical. Counter cyclicality is consistent with the stylized facts of financial emerging markets (see Neumeyer and Perri (2004), and Uribe and Yue (2003)).

5 Contagion

This section characterizes the role of other economies fundamentals in the determination of default incentives to default. As discussed in the introduction there are two channels through which contagion can operate in this framework, one is the wealth channel of contagion and the other is the recomposition channel.

Whenever there is a default by some country in the current period, the wealth channel would propagate this shock while the portfolio recomposition channel will amplify the effect. However when the changes in one country don’t imply an actual default but instead an increase in the probability of default of this country, the wealth channel would still tend to propagate such shock, but the portfolio recomposition channel might be going in an opposite direction.

The reason why the recomposition channel might be going in to an opposite direction than the wealth channel is that a priori assets of different countries are substitutes from the perspective of international investors. However the existence of the wealth channel generates some positive correlation between the return of assets that would be otherwise uncorrelated. If this correlation is not strong enough (i.e., investor’s wealth is pretty high, or they are not too risk averse) because of the recomposition channel it would be observed a "flight to quality" phenomena. That is, countries with better fundamentals will be receiving
more capital flows after a crisis in some emerging economy $k$. In the other hand, if either the wealth of the investors’ is low, or the expected wealth shock is large (i.e., the exposure of the investor to the country with problems is large), or the investor’s risk aversion is high, then the portfolio recomposition channel will be amplifying the effects of the wealth channel due to the increase in the risk of some country $k$.

The following corollaries establish this results formally.

**Default Sets and Other Emerging Countries Default/Repayment State** Default sets also depend on the default/repayment decisions of other economies in investors’ portfolios.

**Corollary 4** There is a wealth channel of contagion. Proposition 3 implies that if economy $k$ defaults in her debts, incentives to default for economy $j$ increase.

**Proof.** See appendix. ■

The intuition of this result is straight forward: a default by some emerging economy in the investors’ portfolio is equivalent to a negative wealth shock. Therefore, from Proposition 3 incentives to default for other economies in the investors’ portfolio increase as a consequence of the default by economy $k$. The result in Corollary 4 is the foundation for the endogenous explanation of contagion of financial crises based on endogenous links across economies that share investors. This result corresponds to the Wealth Channel of contagion.

This result also implies that there is an expected negative wealth effect whenever the probability of default increases for some emerging country in the investors’ portfolio. This wealth effect would tend to reduce the long-term benefits of maintaining access to international credit markets, and therefore would tend to increase the incentives to default for the other countries in the investors’ portfolio.

However whenever the probability of default of some country increases there is also a substitution effect of such change: an increase in the riskiness of some country would tend to induce a substitution in the investors’ portfolio between assets of that country and assets of other emerging countries. This effect is consistent with the evidence regarding the phenomena of “flight to quality” observed during some periods of crises.\(^{15}\)

---

\(^{15}\) According to Kaminsky, Lyons, and Schmukler (2001) during the two first quarters after the Mexican crisis mutual fund flows to countries like Malaysia, Colombia, Poland and Czech Republic increased by more than 10%. During the two first quarters after the Thai crisis, mutual funds flows to countries like Venezuela,
Overall, if the wealth effect dominates the substitution effect it would be reasonable to observe an increase positive correlation across emerging economies sovereign bond prices and capital flows during periods of crises. The numerical analysis of this model suggest that this is the case.

**Corollary 5** There is a recomposition channel of contagion. For the case in which the investor is a risk averse agent, if the overall risk of the investor’s portfolio increases due to the increase in the risk of a particular country, then the incentives to default for all emerging economies increase.

**Proof.** See Appendix.

The intuition for this result comes from the fact that risk averse agents require an increasing risk premium in order to accept a higher level of risk. Therefore whenever their portfolio as a whole becomes more risky investors would ask for a larger risk premium from all the economies in order to allocate the same amount of resources to those countries as before. As a consequence, an increase in risk would reduce the benefits for all the economies to maintain participation in the credit markets, and would therefore increase incentives to default. In turn the overall risk of the investor’s portfolio will increase even further.

As, discussed previously, whenever there is an increase in the probability of default of some emerging economy k, whether or not the overall risk of the investor’s portfolio increases will depend on whether the wealth or the substitution effect dominates. The next corollary establishes this result formally.

**Corollary 6** When the probability of default of some country k increases there is contagion of crises if the expected wealth effect of this change dominates the substitution effect of it. Otherwise, it would be observed a flight to quality type of effect after the fundamentals of economy k deteriorate.

**Proof.** See Appendix.

Numerical results of this paper suggest when considering only productivity shocks which are independently distributed across economies in a two-emerging-economy model, a deterioration of the fundamentals of one economy will cause an endogenous worsening of the other emerging economy which share investors.

Slovak Republic and Sri Lanka increased by more than 5%. Finally, during the two first quarters after the Russian crisis, mutual funds flows to Mexico and Singapore increased by more than 5%.
5.1 Default as an equilibrium outcome of the model

In this model, default can be an equilibrium outcome if the emerging economies ever find it optimal to choose \( b'_j \) such that \( D_j \left( b'_j \mid W'(S), \{ s'_k (S) \}_{k=1,k\neq j} \right) \neq 0 \). In other words, to observe default at equilibrium it must hold that beginning from an asset position \( b_j \) such that \( D_j \left( b'_j \mid W'(S), \{ s'_k (S) \}_{k=1,k\neq j} \right) = 0 \), then there exists a sequence of endowment shocks such that this economy ends up borrowing \( b'_j \) such that \( D_j \left( b'_j \mid W'(S), \{ s'_k (S) \}_{k=1,k\neq j} \right) \neq 0 \). This outcome is possible only if the emerging economy is able to increase its consumption. Default can be an equilibrium outcome if for \( b'_j = \overline{b}_j \left( W'(S), \psi'(\psi), \{ s'_k (S) \}_{k=1,k\neq j} \right) \), the sign of the derivative is ambiguous depending on the emerging economy’s fundamentals, investors’ characteristics and other economies’ fundamentals:

**Proposition 6** Given \( \overline{b}_j \left( W'(S), \psi'(\psi), \{ s'_k (S) \}_{k=1,k\neq j} \right) \), default at equilibrium is a possible outcome of the time series of this model if for \( b'_j = \overline{b}_j \left( W'(S), \psi'(\psi), \{ s'_k (S) \}_{k=1,k\neq j} \right) \)

\[
\frac{\partial c_j}{\partial b_j} \left( W'(S), \psi'(\psi), \{ s'_k (S) \}_{k=1,k\neq j} \right) = - \frac{\partial}{\partial b_j} \left( q_j \overline{b}_j \left( W'(S), \psi'(\psi), \{ s'_k (S) \}_{k=1,k\neq j} \right) \right) < 0.
\]

In other words, default can be an equilibrium outcome if for \( b'_j = \overline{b}_j \left( W'(S), \psi'(\psi), \{ s'_k (S) \}_{k=1,k\neq j} \right) \), it holds

\[
\frac{\partial}{\partial b_j} W'(S),\psi'(\psi),\{ s'_k (S) \}_{k=1,k\neq j} > 0,
\]

so that by increasing its borrowing, the emerging economy is able to increase its consumption.

The sign of this derivative is ambiguous depending on the emerging economy’s fundamentals, investors’ characteristics and other economies’ fundamentals:

\[
\frac{\partial q_j}{\partial b_j} \left( W'(S), \psi'(\psi), \{ s'_k (S) \}_{k=1,k\neq j} \right) \geq 0
\]

\[
\frac{\partial q_j}{\partial b_j} \left( W'(S), \psi'(\psi), \{ s'_k (S) \}_{k=1,k\neq j} \right) \geq 0
\]

\[
\frac{\partial q_j}{\partial b_j} \left( W'(S), \psi'(\psi), \{ s'_k (S) \}_{k=1,k\neq j} \right) \geq 0
\]

(19)
As in Lizarazo (2005), it is not possible to determine the manner in which the sign of the derivative $\frac{\partial q}{\partial b}$ changes with the level of investors’ wealth or risk aversion. Both larger levels of risk aversion or lower levels of wealth would tend to reduce $\bar{b}_j \left(W' (S), \psi' (\psi), \{s_k' (S)\}_{k=1, k \neq j}^J ; \gamma_L \right)$. On the other hand, larger levels of risk aversion or low levels of investors’ wealth would increase the response of the bond prices to changes in the level of borrowing (i.e., will increase $\frac{\partial q}{\partial b}$). Numerical results of the model establish that other things equal, the observation of default in the time series of the model is more likely when investors’ wealth is relatively low or their risk aversion is relatively high.

This result suggests that roughly speaking the smaller is the equilibrium maximum safe level of borrowing, $\bar{b}_j \left(W' (S), \psi' (\psi), \{s_k' (S)\}_{k=1, k \neq j}^J ; \gamma_L \right)$, the higher is the chance that the derivative $\frac{\partial q}{\partial b}$ turns out to be positive. Intuitively, a smaller $\bar{b}_j \left(W' (S), \psi' (\psi), \{s_k' (S)\}_{k=1, k \neq j}^J ; \gamma_L \right)$ suggests that the economies benefit from maintaining access to credit markets is relatively low. Therefore the difference between being in a repayment state or a default state is small in terms of welfare. So the emerging economy’s cost of defaulting is smaller.

Analogous to Arellano (2003), the properties of the hazard function of the endowment process $\left(\frac{f(y'|y)}{1-F(y'|y)}\right)$ are also important in determining the response of default risk to changes in borrowing by the emerging economy $\left(\frac{\partial d}{\partial b(W'(s))}\right)$.

Finally, due to the endogenous links across economies which share investors, at equilibrium both the response of bond prices to borrowing and the maximum safe level of debt are also a function of the fundamentals of all other economies in the investors’ portfolio. In general, for an economy not in default, the maximum safe level of borrowing is smaller whenever another economy is in default; and therefore default is more likely whenever other economies are in default. Furthermore, numerical results of the model establish that default events are more likely whenever the fundamentals of other economies deteriorate.

### 6 Numerical Solution

The model in this paper is not calibrated to match the business cycle statistics of any particular developing country. Instead the model is solved for two hypothetical typical emerging economies. These economies are identical except for the realizations of their endowments. The stochastic process of the endowments is assumed to follow a persistent Markov process. The numerical exercise is performed quarterly. The model parameters are chosen to replicate some features of both typical emerging economies, and typical international investors in
Emerging economies.\textsuperscript{16} Table 1 shows the parameters considered in the numerical analysis of the model.

For the benchmark calibration, the emerging economies’ coefficient of risk aversion is 5, a standard value considered in business cycle literature. As in the previous chapter, the representative investor’s coefficient of risk aversion is set at 0.5. The mean income of the emerging economy is normalized to 1. The representative investor, on the other hand, receives a deterministic income of 0.05 (or 5\% of the emerging economy’s mean income) in each period.\textsuperscript{17} This parameter is chosen so that the equilibrium wealth level of the representative investor is consistent with observed values of the average asset positions of international mutual funds specialized in emerging economies.\textsuperscript{18} Total Net Assets of US Mutual funds for the period 1994-1999 are taken from the “2004 Mutual Fund Book”, published by Investment Company Institute. For mutual funds investing in emerging markets in Latin America, Europe, and Asia, during 1994-1999, the average annual net asset position is US$196.2 Bn (94.3\% of the average emerging economy GDP).\textsuperscript{19}

\begin{table}[h]
\centering
\caption{Contagion: Parameter Values}
\begin{tabular}{|l|c|}
\hline
Parameter & Value \\
\hline
Emerging Economy’s Risk Aversion $\gamma$ & 5 \\
Representative investor’s Risk Aversion $\gamma^L$ & 0.5 \\
Emerging Economy’s Mean Income $E[y]$ & 1 \\
Std. Dev. Emerging Economy’s Income $\text{std}[y]$ & 0.065 \\
Autocorr. Emerging Economy’s Income Process & 0.65 \\
Emerging Economy’s Discount Factor $\beta$ & 0.89 \\
Representative Investor’s Discount Factor $\beta^L$ & 0.9875 \\
Risk Free Interest Rate $r^f = \frac{1}{q^f}$ & 0.01 \\
Representative investor’s Income $X$ & 0.05 \\
\hline
\end{tabular}
\end{table}

\textsuperscript{16}Countries which might be considered similar to the “typical” emerging economy include Argentina, Brazil, Chile, China, Colombia, Czech Republic, Hong Kong, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Russia, Singapore, Slovak, Sri Lanka, Taiwan, Thailand and Venezuela.

\textsuperscript{17}The parameter $X$ is important in the current model. This parameter is the main determinant of the natural credit limit faced by international investors, i.e., the no-ponzi condition. The larger this parameter is, the looser the credit limit, and the wealthier are the investors; consequently when this parameter is larger, the smaller is the impact of changes in wealth over the optimal investors’ portfolio.

\textsuperscript{18}More specifically, ‘average asset positions’ here refer to net asset position as a proportion of the “average” emerging economy GDP.

\textsuperscript{19}Annual average emerging market GDP is computed using the “Global Development Finance and World Development Indicators” of the World Bank. This number takes the average GDP of 25 emerging countries in which mutual funds invested for the period 1994-1999.
The standard deviation of the income process for the emerging economy’s endowment is set to 6.5%. This value is close to the standard deviation of the tradable sector in Argentina but somewhat higher than the variability of the GDP for most emerging markets. The auto-correlation of the endowment process is assumed to be 0.65, roughly the value seen in Brazil, Peru, Slovak Republic and Turkey.

The free interest rate is set to 1%, to match the quarterly US interest rate. The representative investor’s discount factor is set to 0.9875 which is in the range commonly used in business cycle studies of industrialized countries. The emerging economy’s discount rate is chosen to allow the model to exhibit default as an equilibrium outcome of the time series of the model.

The discount factor of the emerging economy is set to 0.89. As discussed in the previous chapter, the overall business cycle statistics are better approximated by using a larger discount rate (0.95), but methodological issues necessitate a relatively small discount rate: First, in order to capture significant dynamics in the present class of models, it is necessary for the economies to have a fine grid in the asset positions due to the steepness of the economies’ bond price functions; this need for a fine grid is present even when the model is solved using interpolation methods. Second, in the present class of models, large discount rates are associated with large equilibrium levels of borrowing; larger levels of borrowing imply that the range of values for each economy’s asset position is larger. The combination of these two issues imply that the minimum dimension of the grid for the economies’ asset position is larger when using high discount rates than when using low discount rates. As a consequence, in order to observed significant dynamics, the minimum size of the required state space is much larger when using high discount rates for the emerging economies.

Due to the limitation of the low discount rate, the results presented here are mainly illustrative, with the objective being to understand the qualitative implications of the model rather than quantitatively matching the business cycle statistics of the emerging economies.

\[^{20}\text{Arellano (2003) reports the standard deviation of the tradable sector in Argentina at around 5.6%}.\]
\[^{21}\text{Valderrama (2002) reports standard deviations in GDP ranging from 1.6\% for Brazil to 5.7\% for Peru. Emerging countries within this range include Argentina, Brazil, Korea, Mexico, Peru, Thailand, and Turkey.}\]
\[^{22}\text{See Aguiar and Gopinath (May 2004). The average autocorrelation for emerging economies reported in that paper is 0.73. By choosing a value below average (but within in the range of observed values), the relatively high variability of the income process is offset.}\]
\[^{23}\text{the size of the state space of the system is given by } n^2 \times s_y^2 \times n_l^2 \text{ where } n \text{ is the size of the grid for the asset position for each economy, } s_y \text{ is the size of the grid for the endowment realization for each economy, and } n_l \text{ is the size of the grid for the investor’s wealth.}\]
6.1 Solution Method

This model is solved numerically using a 'piece-wise' interpolation procedure. The procedure interpolates between discrete values in order to provide a solution to the model which is continuous in \( b \) and \( W \).

The methodology proceeds as follows. Initially, the state space of the model is discretized for each of the state variables of the model, \( b_1, b_2, y_1, y_2 \) and \( W \). The continuous stochastic endowment process of both economies is approximated with a discrete Markov chain that allows for 3 possible realizations of the original process distribution. This approximation is done using the methodology of Hussey and Tauchen (1991). For each emerging economies debt position, \( b_1 \) and \( b_2 \), the asset space takes 30 possible discrete values. Finally, investors’ wealth, \( W \), takes 7 possible discrete values. By interpolating over the grid points, the solution algorithm allows a de facto continuous range for both \( b_1 \), \( b_2 \) and \( W \).

Countries are identical except for the realization of their endowment, therefore the problem is symmetric. Because of the symmetry of the problem it is only necessary to solve the maximization problem of one country. The solution algorithm has the following steps:

(i) Make an initial guess for both emerging economies value function, \( V_j^0 (S) \), next period asset position, \( b_j^0 (S) \), default/repayment decision \( d_j^0 (S) \) and equilibrium price function \( q_j^{APC,0} (S) \), \( j = 1, 2 \). The initial guesses are the value function, the policy function functions and the equilibrium price function that result from an analogous model with risk neutral investors \( V_{RN,0} (S), b_{RN,0} (S), d_{RN,0} (S) \) and \( q_{RN,0} (y; b') \) respectively.

(ii) Taking \( b_j^{*,(i)} (S), d_j^{*,(i)} (S) \) and \( q_j^{APC^{(-i)}} (S) \) as given for \( j = 1, 2 \), and assuming equilibrium in emerging credit markets, that is

\[
\theta_j^{*,(i)} (S) = \begin{cases} 
  b_j^{*,(i)} (S), & \text{if } b_j^{*,(i)} (S) < 0 \\
  0, & \text{if } b_j^{*,(i)} (S) \geq 0 
\end{cases}
\]

iterate on the representative investor’s Bellman equation (4) to solve for the optimal value function \( V_j^{L,i} (S) \) and the optimal policy functions \( W_j^{*,(i)} (S) \)

(iii) Iterate on the emerging economy \( j = 1 \) Bellman equation (12) to solve for the optimal value function \( V_1^{(i)} (S) \), the optimal policy functions \( b_1^{*,(i)} (S) \) and \( d_1^{*,(i)} (S) \) and the corresponding equilibrium price function \( q_1^{EE,(i)} (S; b^{(i)} (S)) \). This iteration involves the next sub-steps:
(a) Take \( q_1^{APC(-i)}(S) \) and \( b_2^{t^*(r(-i)}(S), d_2^{t^*(r(-i)}(S), q_2^{APC(-i)}(S), W^{r,(i)}(S) \) as given to compute \( c_L^{(i)}(S; b') \).

(b) Given \( c_L^{(i)}(S; b') \) and \( b_2^{t^*(r(-i)}(S), d_2^{t^*(r(-i)}(S), q_2^{APC(-i)}(S), W^{r,(i)}(S) \), compute
\[
A_1^{(i)}(S, b_1') =
\beta L \int \left( e^{L'} \right)^{-\gamma} f \left( y_1 | y_1, b_2^{t^*(r(-i)}(S), d_2^{t^*(r(-i)}(S), q_2^{APC(-i)}(S), W^{r,(i)}(S) \right) dy'.
\]

(c) For any \( S, b_1' \) solve for \( q_1^{(i)}(S, b_1') \) by solving the non-linear equation on \( q_1^{(i)}(S, b_1') \) that is derived from (6):
\[
q_1^{(i)}(S, b_1') - b_1' A_1^{(i)}(S, b_1') q_1^{(i)}(S, b_1') - c_L^{(i)}(S; b_1') A_1^{(i)}(S, b_1') = 0
\]
where
\[
c_L^{(i)}(S; b_1', b_2^{t^*(r(-i)}(S), d_2^{t^*(r(-i)}(S), q_2^{APC(-i)}(S)) = X + W - W^{r,(i)} q_1 - d_2^{t^*(r(-i)}(S) b_2^{t^*(r(-i)}(S) - b_1' q_1'.
\]

(d) For any \( S, b_1' \) given \( b_2^{t^*(r(-i)}(S), d_2^{t^*(r(-i)}(S), q_2^{APC(-i)}(S), W^{r,(i)}(S) \) then compute
\[
\beta \int V_1^{C(i)}(S; b_1') f \left( y_1 | y_1, b_2^{t^*(r(-i)}(S), d_2^{t^*(r(-i)}(S), q_2^{APC(-i)}(S), W^{r,(i)}(S) \right) dy'.
\]

(e) Maximize
\[
u \left( y_1 + b_1 - b_1' q_1^{(i)}(S, b_1') \right) + 
\beta \int V_1^{C(i)}(S; b_1') f \left( y_1 | y_1, b_2^{t^*(r(-i)}(S), d_2^{t^*(r(-i)}(S), q_2^{APC(-i)}(S), W^{r,(i)}(S) \right) dy'.
\]
Maximize \( u \left( y_1 + b_1 - b_1' q_1^{(i)}(S, b_1') \right) \) with respect to \( b_1' \) to find \( V_1^{C(i)}(S) \) and the associated \( b_2^{t^*(r(i)}(S) \) and \( q_1^{(i)}(S, b_1^*(i)}(S) \).

(f) Determine \( d_1^{t^*(r(i)}(S) \) by comparing \( V_1^{C(i)}(S) \) to \( V_1^{DP} \).

(g) Determine the equilibrium price of bonds by setting
\[
q_1^{FE(i)}(S; b_1^{t^*(r(i)}(S)) = \begin{cases} q_1^{(i)}(S, b_1^{t^*(r(i)}(S)) & \text{if } d_1^{t^*(r(i)}(S) = 1 \\ 0 & \text{otherwise}. \end{cases}
\]

(h) Set
\[
\begin{align*}
V_2(y_2, y_1, b_2, b_1, W) &= V_1(y_1, y_2, b_1, b_2, W) \\
b_2(y_2, y_1, b_2, b_1, W) &= b_1(y_1, y_2, b_1, b_2, W) \\
d_2(y_2, y_1, b_2, b_1, W) &= d_1(y_1, y_2, b_1, b_2, W) \\
q_2(y_2, y_1, b_2, b_1, W) &= q_1(y_1, y_2, b_1, b_2, W)
\end{align*}
\]
(iv) If \[|q^{EE(i)}(S; b_1^{(i)}(S), b_2^{(i)}(S)) - q^{APC,(-i)}(S; b_1^{(i)}(S), b_2^{(i)}(S))| < \varepsilon\] stop. Otherwise, set \[q^{APC,(-i)}(S; b_1^{(i)}(S), b_2^{(i)}(S)) = q^{EE(i)}(S; b_1^{(i)}(S), b_2^{(i)}(S)),\] and repeat steps 2 to 4.

6.2 Simulations

By allowing for risk aversion on the part of investors and by considering the fundamentals of countries that share investors, the simulations presented here replicate the following observed dynamics of sovereign yield spreads, and capital flows to emerging economies: i) the sovereign risk premium is high during recessions, or when the economy is highly indebted; ii) default is observed when the fundamentals of the economy deteriorate, iii) in periods previous to default the economy experiences capital outflows and collapses in consumption, iv) capital flows and domestic interest rates across emerging economies are positively correlated, and iv) default is observed when the fundamentals of other emerging economies deteriorate.

It is worth noting that the results that follow are derived under the assumption that the punishment for default is a permanent exclusion of the credit market. Therefore the asset distributions of the emerging economy and the investors are degenerate—i.e. as long as default can be a result of the time series, default will occur in finite time in which case both the economy and the investor will remain in permanent autarky. This feature of the model makes the simulations (and therefore the business cycle statistics) highly sensitive to initial conditions. The results shown below assume for initial conditions that each economy begins with its mean income, and zero debt. The statistics shown below are the average for 100 simulations of 100 periods each (i.e. 25 years).

These simulations show that by considering risk averse lenders and multiple countries’ fundamentals, the model developed here provides a better match to both the risk premium of sovereign bond prices as well as to the level of borrowing by emerging economies. Furthermore, because the risk premium in the asset prices has to be large enough to compensate the investor not only for the probability of default, but also for taking the risk of default, the model simulated here is able to account for a larger proportion of credit spreads than models with a representative risk neutral investor.

The business cycle statistics for the benchmark model are given in Table 2.

Table 2 shows that the model reproduces the counter-cyclical behavior of domestic interest rates. In comparing the correlation of domestic interest rates and output between
<table>
<thead>
<tr>
<th></th>
<th>Model with 2 Countries</th>
<th></th>
<th>Model with 1 Country</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean Std Dev Corr Yj</td>
<td>Corr W</td>
<td>Corr Yk</td>
<td>Corr r_k</td>
</tr>
<tr>
<td>Interest Rates</td>
<td>0.031 0.017 -0.052 -0.385 -0.019 0.184</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EE’s Consumption</td>
<td>1.012 0.085 0.801 0.171 0.000 -0.110 1.002 0.097 0.802 0.179</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investor’s Consump.</td>
<td>0.060 0.029 -0.032 0.742 -0.032 -0.220 0.073 0.013 0.032 0.774</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capital Account</td>
<td>0.011 0.037 -0.411 0.334 0.017 -0.460 0.010 0.040 -0.490 0.304</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mean Default Probability (for W > 0)=

- Model with 2 Countries: 0.43
- Model with 1 Country: 0.13

Max Debt Limit=-0.082
the one country and two country models, it can be seen that the correlation falls when a
the two country model is used, going from $-0.233$ to $-0.053$. In principle, this result is
not favorable since the correlation in the two country model is lower than the one country
model and lower than the values reported elsewhere. However, the intuition behind this
correlation suggests that including financial links in sovereign debt models might actually
improve the quantitative performance of these models. To see the potential improvement,
note that when considering a two country model, the probability of observing a default
at equilibrium multiplies by four in comparison to the model with only one country. As
a consequence, with a discount factor of 0.89 default occurs too often and the economy
remains outside of credit markets most of the time. Therefore the domestic interest rate is
relatively insensitive to the economy’s income realizations.

This analysis suggests that introducing financial links might resolve one of the empirical
weakness of models of endogenous sovereign risk. That is, the inclusion of financial links
will allow the use of relatively large values for the discount rate while still observing default
at equilibrium. Using a larger discount rate would imply that a model with the financial
links could support levels of debt which are closer to the observed levels; at the same time
the model would deliver a large risk premium.

In addition to the counter-cyclical behavior of domestic interest rates produced by the
model, Table 2 shows a counter-cyclical relation between the foreign output and the emerg-
ing economies’ domestic interest rates. The value of this correlation is 0.02. As with the
previous case, this number is low; the intuition for this result is analogous to the discussion
of the low countercyclicality of domestic interest rates. Therefore, it is reasonable to expect
that the value of this coefficient can be increased when considering a larger discount rate.
This increase implies a lower default probability, so that bond prices can respond more
to income realizations of the economy. It should be noted that while the correlation of
domestic interest rates and foreign output does not provide a good quantitative fit, from
a qualitative stand point the result is consistent with the empirical literature on conta-
gion: when the fundamentals of foreign emerging economies deteriorate, the spreads of the
domestic emerging economy are affected.

\[24\text{In Lizarazo (2005), the value of the correlation reported is } -0.43. The difference can be explained by}
\[24\text{differences in state space. The process in the current case only allows for three different realizations, while}
\[24\text{the process used in Lizarazo (2005) allows for five values. More values imply a smoother default probability}
\[24\text{function, and therefore a higher sensitivity of the bond price function to income realizations. In other words,}
\[24\text{the pricing function is steeper for the calibration in this article. This steepness implies a lower sensitivity of}
\[24\text{bond prices to the economies’ endowment, and a higher sensitivity to changes in the level of borrowing.}
\[25\text{In Neumeyer and Perri (2004), correlations range from -0.38 for Brazil to -0.7 for Korea, with -0.55 for}
\[25\text{Argentina.} \]
Finally, Table 2 shows the effect that a default by one country has in the premium of the other countries: when 2 countries are in the credit market, the average domestic interest rate is 3.11%. But if one country goes out of the market because it has defaulted, the average domestic interest rate for the remaining country is 3.9%. Table 3 shows the correlations between the prices of bonds of seven emerging economies for the period 1994(Q3)-2000(Q4). The average correlation between the EMBI+ stripped yield spreads of the countries is 0.78 while the correlation explained by this model is 0.18. It is reasonable to expect that considering larger discount rates for the economy might increase the value of this coefficient. With a low value for the emerging economies’ discount rate, the levels of borrowing observed at equilibrium are relatively small. In turn, default risk induces relatively small wealth effects. Another way to improve over this result might be to consider additional fundamental links across economies (i.e. trade links) that can amplify the co-movement of sovereign prices.

Table 4 shows that capital flows to emerging economies are also positively correlated. The average value of this correlation for the period 1994-2004 is 0.43. The model explains a correlation of 0.11 which again is relatively small but positive.

In this model, as in the one country model discussed in Lizarazo(2005), the probability of default increases as the investor’s wealth falls. As before, investors’ wealth is negatively correlated with domestic interest rates. However the correlation between investors wealth and domestic interest rates is reduced when two countries are considered. This reduction might be due to the fact that other countries’ fundamentals affect the domestic interest rates.

---

26 This value is the average of the following countries: Turkey, Czech Republic, Hungary, Russia, South Africa, Costa Rica, Mexico, Argentina, Brazil, Chile, Colombia, Ecuador, Peru, Uruguay, Venezuela, India, China, Hong Kong, Indonesia, Korea, Malaysia, Philippines, Singapore, and Thailand.
Table 4: Debt Securities Issue Abroad 1993(Q3)-2004(Q3)

<table>
<thead>
<tr>
<th>Debt Securities Issued Abroad Correlations</th>
<th>Argentina</th>
<th>Brazil</th>
<th>Mexico</th>
<th>Russia</th>
<th>Philippines</th>
<th>Colombia</th>
<th>Venezuela</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina</td>
<td>1</td>
<td>0.98</td>
<td>0.95</td>
<td>0.94</td>
<td>0.95</td>
<td>0.98</td>
<td>0.88</td>
</tr>
<tr>
<td>Brazil</td>
<td>1</td>
<td>0.93</td>
<td>0.91</td>
<td>0.98</td>
<td>0.99</td>
<td>0.91</td>
<td></td>
</tr>
<tr>
<td>Mexico</td>
<td>1</td>
<td>0.90</td>
<td>0.88</td>
<td>0.90</td>
<td>0.81</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Russia</td>
<td>1</td>
<td>0.91</td>
<td>0.91</td>
<td>0.90</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Philippines</td>
<td>1</td>
<td>0.98</td>
<td>0.94</td>
<td></td>
<td>0.90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Colombia</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>Venezuela</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Joint BIS-IMF-World Bank Statistics on External Debt

In the present model default by one country affects other countries as follows: given the discount rate of 0.89, if there are two countries in the credit market the average capital inflows for each economy are about 1.06% of the GDP. After a default by one country, the average inflows to the other countries are 0.97%.

As with previous models in the literature that do not consider stochastic shocks to the growth trend (i.e., Arellano (2003)), this model cannot match the positive correlation between output and the capital account when considering relatively standard parameters for the volatility of the income process and discount rates. Following Aguiar and Gopinath, the lack of a match in the current paper depends on the large steepness of the price function that results from considering only transitory income shocks. In the current paper, given the level of wealth of international investors, the decision of default is not too sensitive to the realization of income shocks but instead highly sensitive to the quantity borrowed. These interactions result in a very steep equilibrium price function.

Given the steep equilibrium price function, in order to have a counter-cyclical domestic interest rate with transitory income shocks, it must hold that the economy is borrowing more when income is low than when income is high. But more borrowing for low levels of income implies a counter-cyclical current account. This result is counterfactual. However, as in Aguiar and Gopinath (2004), it is reasonable to expect that this model might account for both a counter-cyclical domestic interest rate and a pro-cyclical capital account by considering an endowment process with a permanent component.

Finally, in this model the correlation between foreign emerging economies’ GDP and capital flows to the domestic economy is positive, but small. It is expected that this correlation will increase as the decision to default becomes more sensitive to domestic income realizations. In this case the wealth of the investors will be affected more strongly by the economies’ output realizations; therefore the transmission mechanism of the shocks will be much stronger.
In general, while the results in here are mainly of a qualitative nature, they suggest that the framework presented in this paper can endogenously account for the transmission of crises across emerging economies. Furthermore, the inclusion of financial links across economies might improve the qualitative features of models of endogenous sovereign risk once higher values for the discount rates of the emerging economies are considered.\footnote{The feasibility of undertaking this analysis is under consideration, given the course of dimensionality associated with this model and the high degree of steepness of the bond prices functions.}

7 Conclusions

This paper presents a stochastic dynamic general equilibrium model of default risk that endogenizes the role of external factors in the determination of small open economies’ incentives to default, sovereign bond prices, capital flows and default episodes.

The empirical literature on international finance presents evidence that points to a very relevant role for investors’ characteristics—risk aversion and wealth—and fundamentals of other emerging countries in the determination of sovereign credit spreads and capital flows to emerging economies. The model in this paper is the first model that endogenously determines sovereign bond prices and at the same time endogenously accounts for contagion of crises.

The endogenization of bond prices and contagion occurs in two ways. First, the consideration of enforcement problems in sovereign debt contracts allows default risk and default incentives to be endogenized; therefore sovereign bond prices can be determined endogenously by the model. Second, the assumption of decreasing absolute risk aversion for investors allows for endogenous financial links across economies that share investors. Together, these two elements build a framework that explains the contagion of crisis. The intuition for contagion is as follows: whenever a negative shock occurs in one country, this shock increases the risk associated with that country which implies an expected future negative wealth shocks for investors. Given decreasing absolute risk aversion, investors’ tolerance toward risk decreases following the wealth shock, leading to a portfolio recomposition. Investors shift away from risky investments towards riskless ones.

This explanation for contagion is consistent with the observed behavior of international investors. Investors tend to pull away from other risky countries once one risky country goes into crisis. A testable implication of this explanation would be to test the correlation between the size of investors’ losses and the extent of contagion: if this explanation is
correct, then a larger shock to investors’ wealth should induce a more radical portfolio recomposition away from risky investments.

While the model presented in this chapter provides a strong theoretical explanation for contagion, the implementation of the model faces a hurdle in the limitation that the high dimension of the state space imposes on the numerical solution of the model. This problem might be overcome if the steepness of the pricing function can be reduced. A logical way to do this reduction would be to consider longer term maturities for sovereign bonds. Bonds with longer maturities should have a less steep pricing function since investors would care not only about the next period default risk but future period risk as well. Therefore sovereign bond spreads would be smoother since the spreads might be positive even if there is no chance that the economy will default in the next period. Alternatively, the pricing function can be made less steep by considering permanent income shocks, instead of transitory as in Aguiar and Gophinat (2004).

While the steepness of the bond price function has limited the quantitative results thus far, qualitatively, the results are consistent with the empirical evidence of contagion: First, sovereign bond prices and capital flows to emerging economies are positively correlated across economies. Second, the fundamentals of foreign emerging economies affect the determination of domestic sovereign bond prices and capital flows. Third, the average probability of default for any economy increases when financial links are considered. Finally, the average financing conditions of an economy are less favorable after other countries have defaulted.

References


Appendix

With the exception of the first proposition and its proof, the proofs that follow in this appendix are for the case in which the probability for any emerging economy of coming back to the credit markets after defaulting is 0. Using continuity should be possible to extent them to the case in which this probability is positive.

Proposition 1 For any state of the world, $S$, the endogenous credit constraint $b_j(\{s_k\}_{k=1,k\neq j}^J, W, \psi)$, and the maximum safe level of debt $\overline{b}_j(\{s_k\}_{k=1,k\neq j}^J, W, \psi)$ are singled-valued functions.

Proof. To define these concepts note that the stochastic process for the endowments have a compact support. Also note that, conditional on $W$, the fundamentals of other emerging economies in the investors’ portfolio, and the distribution $\psi$, the value of the credit contract is monotonically decreasing in $b_j$. Monotonicity of the credit contract and compactness of the endowment support are sufficient conditions to guarantee that given the state of the world these critical values (i.e, endogenous credit constraint and maximum safe level of debt) are single-valued functions. ■

Proposition 2 For any state of the world, $S$, as the risk aversion of the international investor increases, the emerging economies’ incentives to default increase.

For $\gamma_L^1 < \gamma_L^2$

$$b_j(\cdot; \gamma_L^2) \geq b_j(\cdot; \gamma_L^1).$$

Proof. The investor’s value function can be written as

$$V^L = E \sum_{t=\tau}^{\infty} \beta^{t-\tau} \left( X + \theta_t^{TB} - q_t^{TB} \theta_{t+1} + \sum_{j=1}^{J} D_{j,t} [\theta_{j,t} - q_{j,t} \theta_{j,t+1}] \right).$$

Assuming an interior solution for the allocation to the emerging economy $j$’s asset

$$\phi(\theta_j') = ED_j \{-q_j v_c (c_L (\theta_j')) + \beta v_c (c_L' (\theta_j')) d_j\} = 0.$$

If the periodic utility of the international investor is of the CRRA type and $\gamma_L^1 < \gamma_L^2$, then there exists a concave function $\kappa(\cdot)$ such that $v_2 (c; \gamma_L^2) = \kappa \left( v_1 (c; \gamma_L^2) \right)$. If $\theta_j'_{1,1}$ is the
optimal allocation when \( \gamma^L = \gamma^L_f \), and \( \theta_{j,2}^f \) is the optimal allocation when \( \gamma^L = \gamma^L_2 \) then it holds that

\[
\begin{align*}
\phi_1 (\theta_{j,1}^f) &= ED_j \left\{ -q_j v_{1,c} (c_L (\theta_{j,1}^f)) + \beta v_{1,c} (c'_{L} (\theta_{j,1}^f)) d_j^f \right\} = 0. \\
\phi_2 (\theta_{j,2}^f) &= ED_j \left\{ -q_j v_{2,c} (c_L (\theta_{j,2}^f)) + \beta v_{2,c} (c'_{L} (\theta_{j,2}^f)) d_j^f \right\} = 0.
\end{align*}
\]

Using \( v_2 (c; \gamma^L_2) = \kappa (v_1 (c; \gamma^L_2)) \) it is possible to define

\[
\phi_2 (\theta_{j,2}^f) = ED_j \kappa' \left\{ v_1 (\theta_{j,1}^f) \right\} \left\{ -q_j v_{1,c} (c_L (\theta_{j,1}^f)) + \beta v_{1,c} (c'_{L} (\theta_{j,1}^f)) d_j^f \right\} < 0.
\]

The last inequality comes from the fact that \( \kappa' (\cdot) \) is positive and decreasing. The inclusion of this function in the previous equation implies that \( \phi_2 (\theta_{j,2}^f) \) is lower than \( \phi_2 (\theta_{j,1}^f) \) because \( \kappa' (\cdot) \) gives little weight to the realizations of \( d_j^f = 1 \), and high weight to the realizations of \( d_j^f = 0 \). Therefore

\[
\phi_2 (\theta_{j,2}^f) > \phi_2 (\theta_{j,1}^f).
\]

The concavity of \( V^L (\cdot) \) implies that given \( q_j \) and the risk of default, \( \phi \left( \theta_{j}^f \right) \), (represented by the expected realizations of \( d_j^f \)) is a decreasing function. As a consequence

\[
\theta_{j,2}^f < \theta_{j,1}^f
\]

which in equilibrium implies \( b_{j,2}^f < b_{j,1}^f \).

Then for any state of the world \( S \), taking as given \( q_j \) and the risk of default \( (\delta_j) \), a higher degree of risk aversion of the investor results in this agent allocating a lower proportion of her portfolio to each of the economies’ sovereign bonds. Therefore, when the investor is less risk averse there are financial contracts that are available to each emerging economy which are not available when the investor is more risk averse. Consequently given \( q_j \) and \( \delta_j \)

\[
V_{1}^C (S; \gamma^L_1) \geq V_{2}^C (S; \gamma^L_2)
\]

Since the utility of autarky for the emerging economies does not depend on the investor’s risk aversion, it is clear that if default is optimal for economy \( j \) when the state of the world is given by \( S \) and \( \gamma^L = \gamma^L_f \), then for the same state of the world \( S \), default would be optimal if \( \gamma^L = \gamma^L_2 \). Additionally, because incentives to default would be higher whenever \( \gamma^L = \gamma^L_f \) than if \( \gamma^L = \gamma^L_1 \), then at equilibrium \( \delta (S, b'; \gamma^L_2) > \delta (S, b'; \gamma^L_1) \). Therefore \( q (S, b'; \gamma^L_2) < q (S, b'; \gamma^L_1) \). In conclusion, for all states of the world incentives to default for each emerging economy become stronger when the investor’s risk aversion is larger.

**Proposition 3** Default sets are shrinking in the assets of the representative investor. For all \( W_1 < W_2 \), if default is optimal for \( b_j \) in some states \( y_j \), given \( W_2 \) then default will be
optimal for $b_j$ for the same states $y_j$, given $W_1$ therefore $D_j \left( b_j \mid W_2, \psi, \{s_k\}_{k=1,k \neq j}^J \right) \subseteq D_j \left( b_j \mid W_1, \psi, \{s_k\}_{k=1,k \neq j}^J \right)$.

For $W_1 < W_2$ it must hold
\[
\frac{b_j}{b_j} \left( W_1, \psi, \{s_k\}_{k=1,k \neq j}^J \right) \geq \frac{b_j}{b_j} \left( W_2, \psi, \{s_k\}_{k=1,k \neq j}^J \right) \geq \frac{s_j}{s_j} \left( W_2, \psi, \{s_k\}_{k=1,k \neq j}^J \right).
\]

Proof. From (6) if $W_1 < W_2$ then for each economy $j$ taking as given $q_j$ and the level of default risk ($\delta_j$)
\[
b_j' < b_j' \subset b_j'.
\]
This inequality holds because decreasing absolute risk aversion implies that $v \left( X + W_1 - q_j \theta_{t+1}^{TB} - \sum_{j=1}^{J} D_j, q_j, \theta_{j,t+1} \right)$ is a concave transformation of $v \left( X + W_2 - q_j \theta_{t+1}^{TB} - \sum_{j=1}^{J} D_j, q_j, \theta_{j,t+1} \right)$ (see Proposition 6.C.3 of Mas-Collel, Whinston, and Green). So if $\theta_{j,1}'$ is the optimal allocation to emerging economy $j$’s assets when $W = W_1$, and $\theta_{j,2}'$ is the optimal allocation to this economy when $W = W_2$, and defining $v_1 (\theta_{j,1,t+1}) = v \left( X + W_1 - q_j \theta_{t+1}^{TB} - \sum_{j=1}^{J} D_j, q_j, \theta_{j,1,t+1} \right)$, and
\[
v_2 (\theta_{j,2,t+1}) = v \left( X + W_2 - q_j \theta_{t+1}^{TB} - \sum_{j=1}^{J} D_j, q_j, \theta_{j,2,t+1} \right) \text{ then}
\]
\[
\phi_1 (\theta_{j,1}') = ED_j \left\{ -q_j v_{1,c} (c_L (\theta_{j,1}')) + \beta v_{1,c} (c_L (\theta_{j,1}')) d_j' \right\} = 0,
\]
\[
\phi_2 (\theta_{j,2}') = ED_j \left\{ -q_j v_{2,c} (c_L (\theta_{j,2}')) + \beta v_{2,c} (c_L (\theta_{j,2}')) d_j' \right\} = 0.
\]
And because $v_1 (c) = \kappa (v_2 (c))$
\[
\phi_1 (\theta_{j,2}') = ED_j \kappa' \left[ v_2 (\theta_{j,2}') \right] \left\{ -q_j v_{2,c} (c_L (\theta_{j,2}')) + \beta v_{2,c} (c_L (\theta_{j,2}')) d_j' \right\} < 0.
\]
The previous inequality comes from the fact that $\kappa' (\cdot)$ is positive and decreasing. The inclusion of this function in the previous equation implies that $\phi_1 (\theta_{j,2}')$ is lower than $\phi_1 (\theta_{j,1}')$ because $\kappa' (\cdot)$ gives little weight to the realizations of $d_j' = 1$, and high weight to the realizations of $d_j' = 0$. Therefore
\[
\phi_1 (\theta_{j,2}') < \phi_1 (\theta_{j,1}').
\]
The concavity of $V^L (\cdot)$ implies that given $q_j$ and the risk of default, $\phi \left( \theta_{j}' \right)$ is a decreasing function, and as consequence
\[
\theta_{j,2}' > \theta_{j,1}'
\]
which in equilibrium implies $b'_{j,2} < b'_{j,1}$.

Because the representative agent of the emerging economy is able to observe the optimal response function of the investors, when $W$ increases this agent modifies her actions to get the best available contract under this state of the world which is given by

$$\left\{ q_j \left(S_2; \left\{ b'_m (S_2) \right\}_{m=1}^J \right), \left\{ b'_m (S_2) \right\}_{m=1}^J \right\}$$

The representative agent of the emerging economy $j$ chooses $b'_j (S_2)$ knowing than the collective action of the investors implies that the equilibrium price of the sovereign bonds is $q \left(S_2; \left\{ b'_m (S_2) \right\}_{m=1}^J \right)$. Then for any given level of bond prices, each emerging economy $j$ is able to borrow at least as much when the wealth of investors is $W_1$ as when the wealth of the investors is $W_2$. When the state of the world is $S_2$, because the representative agent of the emerging economy chooses $\left\{ q (S_2; \left\{ b'_m (S_2) \right\}), \left\{ b'_m (S_2) \right\} \right\}$ even though the financial contract $\left\{ q (S_1; \left\{ b'_m (S_1) \right\}), \left\{ b'_m (S_1) \right\} \right\}$ is available, then it holds that

$$V^c_j (S_2) > V^c_j (S_1)$$

Given that when the state of the world is $S_2$ default is the optimal choice, it must hold

$$V^A_j (y_j) > V^c_j (S_2) > V^c_j (S_1)$$

which implies that if default is optimal for $b_j$ in some states $y_j$, given $W_2$, then default is optimal for the same states given $W_1$.

**Proposition 4** Default sets are shrinking in assets of the emerging economy. For all $b_{j,1} < b_{j,2}$, if default is optimal for $b_{j,2}$ in some states $y_j$, given $W$, and the asset position of other emerging economies in the investors’ portfolio, then default will be optimal for $b_{j,1}$ for the same states $y_j$, given $W$, and the asset position of other emerging economies in the investors’ portfolio. Therefore $D_j \left( b_{j,2} \mid W, \psi, \left\{ s_k \right\}_{k=1,k\neq j} \right) \subseteq D_j \left( b_{j,1} \mid W, \psi, \left\{ s_k \right\}_{k=1,k\neq j} \right).$
If the endowment process is the result in proposition 2, investors’ wealth would be \( \pi_q = b_j^* (S_j) \). That is, the emerging economy

\[
\frac{\pi_q (S_j)}{p_j} \]

always could choose to transfer \( y_j \), while \( p_j \). Since default is optimal for \( b_j^* \) then

\[
u (y + b_j^* - q_j (S_j^*; (S_j^*), (s_j^* (S_j^*))_{k=1,k\neq j}^J, b_j^* (S_j^*)) + \beta E \left[ V \left( b_j^* (S_j^*), (s_j^* (S_j^*))_{k=1,k\neq j}^J, y_j^*, (y_j^*)_{k=1,k\neq j}^J, W^* (S_j^*) \right) \right] > 0
\]

\[
u (y + b_j^* - q_j (S_j^*; (S_j^*), (s_j^* (S_j^*))_{k=1,k\neq j}^J, b_j^* (S_j^*)) + \beta E \left[ V \left( b_j^* (S_j^*), (s_j^* (S_j^*))_{k=1,k\neq j}^J, y_j^*, (y_j^*)_{k=1,k\neq j}^J, W^* (S_j^*) \right) \right] > 0
\]

\[
u (y + b_j^* - q_j (S_j^*; (S_j^*), (s_j^* (S_j^*))_{k=1,k\neq j}^J, b_j^* (S_j^*)) + \beta E \left[ V \left( b_j^* (S_j^*), (s_j^* (S_j^*))_{k=1,k\neq j}^J, y_j^*, (y_j^*)_{k=1,k\neq j}^J, W^* (S_j^*) \right) \right] > 0
\]

\[
u (y + b_j^* - q_j (S_j^*; (S_j^*), (s_j^* (S_j^*))_{k=1,k\neq j}^J, b_j^* (S_j^*)) + \beta E \left[ V \left( b_j^* (S_j^*), (s_j^* (S_j^*))_{k=1,k\neq j}^J, y_j^*, (y_j^*)_{k=1,k\neq j}^J, W^* (S_j^*) \right) \right] > 0
\]

\[
u (y + b_j^* - q_j (S_j^*; (S_j^*), (s_j^* (S_j^*))_{k=1,k\neq j}^J, b_j^* (S_j^*)) + \beta E \left[ V \left( b_j^* (S_j^*), (s_j^* (S_j^*))_{k=1,k\neq j}^J, y_j^*, (y_j^*)_{k=1,k\neq j}^J, W^* (S_j^*) \right) \right] > 0
\]

where \( S_1 = (b_{j,1}, (s_k)_{k=1,k\neq j}^J, y_j, W, \psi) \), \( S_2 = (b_{j,2}, (s_k)_{k=1,k\neq j}^J, y_j, W, \psi) \), and \( \tilde{S}_1 = (b_{j,2}, (s_k)_{k=1,k\neq j}^J, y_j, \tilde{W}, \psi) \), where \( \tilde{W} = \theta^{TB} + (-b_{j,1}) + \sum_{k=1,k\neq j}^J -b_k > \theta^{TB} + (-b_{j,2}) + \sum_{k=1,k\neq j}^J -b_k = W \).

The first inequality holds because given \( b_{j,2} \) the economy \( j \) could choose to borrow \( b_j^* = b_j^* (\tilde{S}_1) \), to obtain a bond price \( \tilde{q}_j (S_j^*; (S_j^*), (s_j^* (S_j^*))_{k=1,k\neq j}^J, b_j^* (S_j^*)) \), such that

\[
\tilde{q}_j (S_j^*; (S_j^*), (s_j^* (S_j^*))_{k=1,k\neq j}^J, b_j^* (S_j^*)) = \tilde{q}_j (\tilde{S}_1; (S_j^*), (s_j^* (S_j^*))_{k=1,k\neq j}^J, b_j^* (S_j^*)) \]

but chooses not to. That is, the emerging economy \( j \) always could choose to transfer \( T = b_{j,2} - b_{j,1} \) to the investors, so that the economy’s cash flow would be \( y_j + b_{j,1} \) instead of \( y_j + b_{j,2} \), and the investors’ wealth would be \( \tilde{W} \) instead of \( W \). The third inequality holds as a consequence of the result in proposition 2.

Since default is optimal for \( b_{j,2} \), it must hold that

\[
V^A (y_j) > V^C (S_j^*) > V^C (S_j)
\]

which implies that for \( b_{j,1} \) default is also optimal in the same states \( y \). ■

Proposition 5 If the endowment process is i.i.d., default incentives are stronger the
lower the endowment. For all \( y_{j,1} < y_{j,2} \) if \( y_{j,2} \in D_j(b_j | W, \psi, \{s_k\}_{k=1,k\neq j}^J) \) then \( y_{j,1} \in D_j(b_j | W, \psi, \{s_k\}_{k=1,k\neq j}^J) \).

**Proof.** Because \( y_{j,2} \in D \left( b_j | W, \psi, \{s_k\}_{k=1,k\neq j}^J \right) \) then \( V^A(y_{j,2}) \geq V^C_j(S_2) \), where

\[
V^C_j(S_2) = u\left(y_{j,2} + b_j - q_j(S_2; b_j'(S_2), \psi) b_j'(S_2)\right) + \beta E\left[V\left(b_j'(S_2), S'(S_2)\right)\right]
\]

A sufficient condition for \( y_{j,1} \in D \left( b_j | W, \psi, \{s_k\}_{k=1,k\neq j}^J \right) \) is given by

\[
0 \geq V^C_j(S_2) - V^A(y_{j,2}) > V^C_j(S_1) - V^A(y_{j,1}) \quad \text{(A-1)}
\]

In state \( S_2 \) the representative agent of the emerging economy can choose to destroy a portion \( T = y_{j,2} - y_{j,1} \) of her endowment. If that were the case the state of the world would be \( S(b_j, \{s_k\}_{k=1,k\neq j}^J, y_2 - T, W, \psi) \) which is simply \( S_1 \). In this case economy \( j \) would optimally enter the contract \( \left\{ b_j'(S), q_j(S; b_j'(S)), \{b_k'(S)\}_{k=1,k\neq j}^J \right\} \) which corresponds to \( \left\{ b_j'(S_1), q_j(S; b_j'(S_1)), \{b_k'(S_1)\}_{k=1,k\neq j}^J \right\} \). Therefore the contract \( \left\{ b_j'(S_1), q_j(S_1; b_j'(S_1), \{b_k'(S_1)\}_{k=1,k\neq j}^J \right\} \) is in the in the emerging economy \( j \)'s possibility set when the state of the world is given by \( S_2 \). Utility maximization implies

\[
V^C_j(S_2) \geq u\left(y_{j,2} + b_j - q_j(S_1; b_j'(S_1), \{b_k'(S_1)\}_{k=1,k\neq j}^J) b_j'(S_1)\right) + \beta E\left[V\left(b_j'(S_1), \{b_k'(S_1)\}_{k=1,k\neq j}^J, y_j', \{y_k'\}_{k=1,k\neq j}^J, W'(S_1)\right)\right]
\]

If

\[
u\left(y_{j,2} + b_j - q_j(S_1; b_j'(S_1), \{b_k'(S_1)\}_{k=1,k\neq j}^J) b_j'(S_1)\right) + \beta E\left[V\left(b_j'(S_1), \{b_k'(S_1)\}_{k=1,k\neq j}^J, y_j', \{y_k'\}_{k=1,k\neq j}^J, W'(S_1)\right)\right] - V^C_j(S_1) \quad \text{(A-2)}
\]

then by transitivity \((A-1)\) holds. For the \( i.i.d. \) case, \((A-2)\) holds if and only if

\[
u\left(y_{j,2} + b_j - q_j(S_1; b_j'(S_1), \{b_k'(S_1)\}_{k=1,k\neq j}^J) b_j'(S_1)\right) - u\left(y_{j,1} + b_j - q_j(S_1; b_j'(S_1), \{b_k'(S_1)\}_{k=1,k\neq j}^J) b_j'(S_1)\right) > V^A(y_{j,2}) - V^A(y_{j,1}) \quad \text{(A-3)}
\]

Because the contract \( \left\{ b_j'(S_1), q_j(S_1; b_j'(S_1), \{b_k'(S_1)\}_{k=1,k\neq j}^J \right\} \) belongs to the possibility set for the emerging economy when the state of the world is \( S_2 \), and since for this state of
the world economy $j$ finds it optimal to default (i.e., \( y_{j,2} \in D \left( b_j | W, \psi, \{ s_k \}_{k=1,k\neq j}^J \right) \)), then it must hold \( b_j - q_j \left( s_1; b_j^l (S_1), \{ b_k^l (S_1) \}_{k=1,k\neq j}^J \right) < 0 \). Given that \( u(\cdot) \) is increasing in \( y_j \), and strictly concave \((A - 3)\) holds, therefore \( y_{j,1} \in D \left( b_j | W, \psi, \{ s_k \}_{k=1,k\neq j}^J \right) \).

**Corollary A1** There is a wealth channel of contagion. Proposition 3 implies that if economy $k$ defaults in her debts, incentives to default for economy $j$ increase.

**Proof.** If economy $k$ defaults in her debts with the investor, the wealth of this agent will be \( W | d_k = 0 = \theta^{TB} + \sum_{m=1,m\neq k}^J \theta_m' \), which is lower than the wealth for economy $k$ if she decides not to default, which is \( W | d_k = 1 = \theta^{TB} + \sum_{m=1,m\neq k}^J \theta_m' + \theta_k' \). Therefore

\[
V^C_j \left( y_j, b_j, \{ y_m \}_{m=1}^J, \{ b_m \}_{m=1}^J, (W | d_k = 1) \right) > V^C_j \left( y_j, b_j, \{ y_m \}_{m=1}^J, \{ b_m \}_{m=1}^J, (W | d_k = 0) \right)
\]

which implies that emerging economy $j$’s incentives to default are larger when some economy $k$ which shares investors defaults.

**Corollary A2** There is a recomposition channel of contagion. For the case in which the investor is a risk averse agent, if the overall risk of the investor’s portfolio increases due to the increase in the risk of a particular country, then the incentives to default for all emerging economies increase.

**Proof.** The investor’s value function can be written as

\[
V^L = E \sum_{t=\tau}^{\infty} \beta^{t-\tau} v \left( X + \theta_t^{TB} - q_j \theta_{t+1}^{TB} + \sum_{j=1}^J D_{j,t} [\theta_{j,t} - q_{j,t} \theta_{j,t+1}] \right)
\]

Assuming an interior solution for the allocation to the emerging economy $j$’s asset

\[
\phi \left( \theta_j' \right) = ED_j \left\{ -q_j v_c (c_L (\theta_j')) + \beta v_c (c'_L (\theta_j')) d_j' \right\} = 0.
\]

Taking the risk of economy $j$ as given, compare this first order condition for two scenarios: one in which the risk of economy $k$ is relatively small such that \( E d_k' = 1 - \delta_L \) versus one in which the risk of economy $k$ is relatively high such that \( E d_k' = 1 - \delta_H \), with \( \delta_H > \delta_L \). Then it holds that

\[
\phi_L \left( \theta_{j,L} \right) = E_L \left[ D_j \left\{ -q_j v_c (c_L (\theta_{j,L})) + \beta v_c (c'_L (\theta_{j,L})) d_j' \right\} \right] = 0.
\]

\[
\phi_H \left( \theta_{j,H} \right) = E_H \left[ D_j \left\{ -q_j v_c (c_L (\theta_{j,H})) + \beta v_c (c'_L (\theta_{j,H})) d_j' \right\} \right] = 0.
\]

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In this case, if the increase in the risk of emerging economy $k$ generates an increase in the overall risk of the investor’s portfolio, then it must hold

$$\phi_H (\theta'_{j,L}) = E_i H \left[ D_j \left\{ -q_j v_c (c_L (\theta'_{j,L})) + \beta v_c (c'_L (\theta'_{j,L})) \right\} d'_j \right] > 0. \quad (A-4)$$

The previous inequality holds because an increase in the overall risk of the portfolio implies

$$E_i H \left[ v_c (c'_L (\theta'_{j,L})) d'_j \right] = \int [v_c (c'_L (\theta'_{j,L})) d'_j] d (d'_j | d_{k,H}) > \int [v_c (c'_L (\theta'_{j,L})) d'_j] d (d'_j | d_{k,L}) = E_i L \left[ v_c (c'_L (\theta'_{j,L})) d'_j \right]$$

Equation (A - 4) implies that

$$\phi_H (\theta'_{j,L}) > \phi_H (\theta'_{j,H})$$

The concavity of $V^L (\cdot)$ implies that given $q_j$, and $d_j \phi (\theta'_j)$ is a decreasing function, and as consequence

$$\theta'_{j,H} < \theta'_{j,L}$$

which in equilibrium implies $b'_{j,H} > b'_{j,L}$.

Then for any state of the world $S$, taking as given $q_j$, and $d_j$, a higher overall risk in the investor’s portfolios would result in the investor allocating a lower proportion of her portfolio to economy $j$’s sovereign bonds. Therefore, when the overall risk of the investor’s portfolio is low there are financial contracts that are available to economy $j$ that are not available when the risk of this portfolio is larger. Consequently given $q_j$

$$V^{C,C}_{j,1} (S_L) \geq V^{C,C}_{j,2} (S_H)$$

Because the utility of autarky for the emerging economies does not depend on the risk of other economies different to $j$, it is clear that if default is optimal for economy $j$ if the state of the world is given by $S_L$, then default is also optimal for economy $j$ when the state of the world is given by $S_H$. This holds true for any economy $j$, $j = 1, 2, \ldots, J$, $j \neq k$.

To summarize, the incentives to default for all emerging economies become stronger when an increase in the risk of default of economy $k$ causes and increase in overall risk of the investor’s portfolio. □

**Corollary A3** When the probability of default of some country $k$ increases contagion of crises occurs if the expected wealth effect of this change dominates the substitution effect. Otherwise, a “flight to quality” type of effect would be observed.

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Proof. If \( \delta_{k,1} > \delta_{k,2} \) then

\[
EW' (S; \delta_{k,1}) < EW' (S; \delta_{k,2})
\]

which implies

\[
E \left[ V \left( b'_j, \{ y'_m \}_{m=1,m\neq j} \right), y'_j, \{ y'_m \}_{m=1,m\neq j}, W' (S; \delta_{k,1}) \right] < E \left[ V \left( b'_j, \{ y'_m \}_{m=1,m\neq j} \right), y'_j, \{ y'_m \}_{m=1,m\neq j}, W' (S; \delta_{k,2}) \right].
\]

On the other hand,

\[
u \left( y_j + b_j - q_j \left( S; b'_j, \{ y'_m \}_{m=1,m\neq j} | \delta_{k,1} \right) b'_j \right) \leq u \left( y_j + b_j - q_j \left( S; b'_j, \{ y'_m \}_{m=1,m\neq j} | \delta_{k,2} \right) b'_j \right)
\]

which is true because, if assets of different countries are substitutes, then investors demand more assets of other countries whenever country \( k \) becomes more risky. This effect leans toward the direction of having

\[
q_j \left( S; b'_j, \{ y'_m \}_{m=1,m\neq j} | \delta_{k,1} \right) > q_j \left( S; b'_j, \{ y'_m \}_{m=1,m\neq j} | \delta_{k,2} \right).
\]

On the other hand, the expected wealth effect of changes in the country \( k \) default probability lean toward the direction of having

\[
q_j \left( S; b'_j, \{ y'_m \}_{m=1,m\neq j} | \delta_{k,1} \right) < q_j \left( S; b'_j, \{ y'_m \}_{m=1,m\neq j} | \delta_{k,2} \right).
\]

If the substitution effect is large enough

\[
V^C_j (\cdot, \delta_{k,1}) = u \left( y_j + b_j - q_j \left( S; b'_j, \{ y'_m \}_{m=1,m\neq j} | \delta_{k,1} \right) b'_j \right)
+ \beta E \left[ V \left( b'_j, \{ y'_m \}_{m=1,m\neq j}, y'_j, \{ y'_m \}_{m=1,m\neq j}, W' (S; \delta_{k,1}) \right) \right]
\]

\[
> u \left( y_j + b_j - q_j \left( S; b'_j, \{ y'_m \}_{m=1,m\neq j} | \delta_{k,2} \right) b'_j \right)
+ \beta E \left[ V \left( b'_j, \{ y'_m \}_{m=1,m\neq j}, y'_j, \{ y'_m \}_{m=1,m\neq j}, W' (S; \delta_{k,2}) \right) \right] = V^C_j (\cdot, \delta_{k,2}).
\]

Otherwise

\[
V^C_j (\cdot, \delta_{k,1}) < V^C_j (\cdot, \delta_{k,2}).
\]

In the latter case, when the fundamentals of other emerging economies deteriorate, contagion occurs. \( \blacksquare \)

Proposition 6 Given \( b'_j \left( W' (S) , \{ b'_{k} (S) \}_{k=1,k\neq j} \right) \), default at equilibrium is a possible outcome of the time series of this model if for \( b'_j = b'_j \left( W' (S) , \{ b'_{k} (S) \}_{k=1,k\neq j} \right) \)
\[ \frac{\partial c_j}{\partial y_j} \left( W'(S), \{b'_k(S)\}_{k=1,k\neq j}^J \right) = - \frac{\partial \left( q_j b_j \left( W'(S), \{b'_k(S)\}_{k=1,k\neq j}^J \right) \right)}{\partial y_j} \left( W'(S), \{b'_k(S)\}_{k=1,k\neq j}^J \right) < 0. \]

In other words, default can be an equilibrium outcome if for \( b'_j = b_j \left( W'(S), \{b'_k(S)\}_{k=1,k\neq j}^J \right) \), it holds that

\[ \frac{\partial q_j b_j W'(S), \{b'_k(S)\}_{k=1,k\neq j}^J}{\partial y_j} > 0, \]

so that by increasing its borrowing, the emerging economy is able to increase its consumption.

**Proof.** For default to be a possibility, it must be true that it is optimal for the economy to borrow beyond its maximum safe level of debt, so that for some \( b'_j \left( W'(S), \{b'_k(S)\}_{k=1,k\neq j}^J \right) < b_j \left( W'(S), \{b'_k(S)\}_{k=1,k\neq j}^J \right) \)

such that

\[ \delta_j \left( y_j, \{y_k\}_{k=1,k\neq j}^J, b'_j, \{b'_k(S)\}_{k=1,k\neq j}^J, W'(S) \right) > \delta_j \left( y_j, \{y_k\}_{k=1,k\neq j}^J, b_j, \{b'_k(S)\}_{k=1,k\neq j}^J, W'(S) \right) = 0, \]

it holds that

\[ u \left( y_j + b_j - q_j \left( S; b'_j(S), \{b'_k(S)\}_{k=1,k\neq j}^J \right) b'_j \left( W'(S), \{b'_k(S)\}_{k=1,k\neq j}^J \right) \right) + \beta E \left[ V \left( b'_j(S), \{b'_k(S)\}_{k=1,k\neq j}^J, y'_j, \{y'_k\}_{k=1,k\neq j}^J, W'(S) \right) \right] > u \left( y_j + b_j - q_j \left( S; b_j(S), \{b'_k(S)\}_{k=1,k\neq j}^J \right) b_j \left( W'(S), \{b'_k(S)\}_{k=1,k\neq j}^J \right) \right) + E \left[ V \left( b_j(S), \{b'_k(S)\}_{k=1,k\neq j}^J, y'_j, \{y'_k\}_{k=1,k\neq j}^J, W'(S) \right) \right]. \]  

(A-5)

But by Proposition ??

\[ \beta E \left[ V \left( b'_j(S), \{b'_k(S)\}_{k=1,k\neq j}^J, y'_j, \{y'_k\}_{k=1,k\neq j}^J, W'(S) \right) \right] < \beta E \left[ V \left( b_j(S), \{b'_k(S)\}_{k=1,k\neq j}^J, y'_j, \{y'_k\}_{k=1,k\neq j}^J, W'(S) \right) \right]. \]

Therefore, in order for (A - 5) to hold, it must be true that

\[ u \left( y_j + b_j - q_j \left( S; b'_j(S), \{b'_k(S)\}_{k=1,k\neq j}^J \right) b'_j \left( W'(S), \{b'_k(S)\}_{k=1,k\neq j}^J \right) \right) \]

(A-6)

which implies that

\[ \frac{\partial c_j}{\partial y_j} \left( W'(S), \{b'_k(S)\}_{k=1,k\neq j}^J \right) = - \frac{\partial \left( q_j b_j \left( W'(S), \{b'_k(S)\}_{k=1,k\neq j}^J \right) \right)}{\partial y_j} \left( W'(S), \{b'_k(S)\}_{k=1,k\neq j}^J \right) < 0. \]
A APPENDIX

B Literature Survey

In the case of time clustered crises, the issue of distinguishing between common factors affecting several countries at once versus contagion is completely an empirical question; examples of contagion were given in the previous section of this paper. On the other hand, once contagion has been identified, explaining why contagion occurs is both a theoretical and an empirical question. From a theoretical point of view, three main types of economic linkages across countries are considered to explain contagion of crisis: a) trade links, b) financial links, and c) links based on the informational structure of financial markets and the failure of coordination between agents. The empirical strand of the literature is mainly concerned with determining the relative importance of each of these channels of contagion.

Trade Links

Theoretical Literature The theoretical literature on contagion through trade links argues that a crisis in one country will have a negative impact on the economy of any commercial partner of that nation as follows. A crisis in one country generates a contraction of aggregate demand in that economy. This contraction can have severe income effects for countries that export to the crisis country. A couple of papers consider this channel of contagion within a general equilibrium framework. Diao and Yeldan (2000) calibrate a deterministic multi-country model for the case of the Asian crisis. The authors find some role for trade links in explaining crises in the Asian region. In a more theoretical paper, Paasche (2001) analyzes the role of commercial links between countries in explaining contagion. The author concludes that only in the presence of financial market imperfections can shocks in one country lead to significant capital outflows and rapid deterioration in the current account of the trade partner.

Corsetti, Presenti, Roubini, and Tille (1998) explain contagion of a currency crisis through currency devaluation in a crisis country. The devaluation channel operates between commercial competitors. The economic intuition behind this channel is that in a world with some degree of nominal rigidities, a crisis which generates a nominal devaluation of the domestic currency in one country can have a negative impact on the international competitiveness of other countries. This impact arises as the devaluation changes the relative prices of the countries’ goods in world markets. The resulting loss of competitiveness
of commercial competitors of the crisis country might generate severe declines in the export revenues of those countries. The devaluation channel for contagion will not be considered in the present work.

**Empirical Literature**  The empirical evidence on the relative importance of trade links in the transmission of crisis is mixed. Several studies find evidence that trade links do not play a significant role in explaining contagion. Examples of these studies include Kaminsky and Reinhart (1998) for the case of the Mexican and Asian crises, and Baig and Goldfajn (1998) for the case of the Asian Crises. Several other studies find evidence that trade linkages are an important channel of contagion. Examples of these studies include Eichengreen, Rose and Wyplosz (1996), De Gregorio and Valdes (2000), Hernandez, Mellado and Valdes (2001), and Forbes (2000). However, none of these studies control for financial links. Given the difficulty in separating financial links and trade links—empirically trade and financial flows are highly correlated—it is not possible to ignore the possibility that the results in these studies might be strongly affected by omitted variable bias. The only empirical support for trade linkages which is not subject to omitted variable bias is Forbes (2001). However, this study finds that trade links were able to explain at most 1/4 of the variation in stock market returns during crisis periods.

**Financial Links**

**Theoretical Literature**  There are three types of financial links that can explain contagion.

First, a crisis can spread across countries through a *liquidity channel* when a crisis in one country or region affects the liquidity position of lenders/investors exposed to that country. This exposure forces investors to recall investments in other countries or regions in order to meet redemptions in the crisis country. There is a conceptual problem with

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28 Kaminsky and Reinhart (1998) report that during the Mexican and the Asian Crises bilateral and third party trade links with the ground-zero countries (Mexico and Thailand respectively) were weak in the case of the most affected countries in each crisis (Argentina and Indonesia respectively).

For the case of the Asian crises, Baig and Goldfajn (1998) argue that it is difficult to explain contagion as resulting from either direct or indirect trade links: for the Asian region, the average share of exports to Thailand, the ground-zero country, was less than 4% of the total exports. Furthermore, eventhough the five affected countries exported a large portion of their goods to US and Japan, since the export patterns were highly differentiated, it is difficult to make a case for the competition effect.

29 Forbes (2001) uses industry level data. While trade is highly differentiated across industries, financial flows are not, so the high degree of correlation of trade flows and financial flows is broken.
considering the liquidity channel as the only source of contagion: unless lenders are banned from investment opportunities in developed countries, then developed countries should be the most vulnerable to contagion. This vulnerability would arise because selling in highly liquid and deeper markets would lower the impact that sell orders have on asset prices and allow lenders to avoid further capital losses.

The liquidity channel is considered in several papers. Using a stochastic dynamic general equilibrium framework, Valdes (1996) and Allen and Gale (2000) develop 3-period models with exogenously given technology for liquidation of long term assets. In Valdes (1996), the contagion outcome is highly dependent on the order in which assets are liquidated by investors after an exogenous liquidity shock hits those agents. In Allen and Gale (2000) contagion is only a possibility if completely unanticipated shocks occur and if banks of different regions or countries choose to insure against liquidity shocks through an interbank market of deposits. If instead banks are organized to protect themselves from liquidity shocks through an interbank market of ex-post loans or if liquidity shocks are anticipated, contagion is not possible. Miller (1998) employs a deterministic two country representative agent model that also considers the liquidity channel, but the model does not have a theory for the origin of the crisis—i.e. the original crisis is a consequence of self-fulfilling expectations. Therefore the model suffers from a problem that will be discussed later on for multiple equilibria models.

Second, a crisis can spread across countries through a wealth channel when a crisis in one country spreads to other countries through the deterioration in the portfolio of investors exposed to the crisis country. Depending on the lenders’ preferences, portfolio deterioration might reduce investors’ risk tolerance and lead them to reduce the degree of exposure to other risky investments in an attempt to reduce the overall risk of their portfolios.

The wealth channel is considered in several papers. Goldstein and Pauzer (2001) and Kyle and Xiong (2001) consider this link within stochastic general equilibrium models. Goldstein and Pauzer (2001) analyze contagion based on wealth effects in a two-period model of a world with multiple equilibria. In this world it is possible to pin down the new equilibria after a initial crisis because agents make their portfolio choices sequentially. In this paper, when investors realize financial losses, the reduction in their tolerance toward risk derives from their preferences—investors’ preferences exhibit decreasing absolute risk aversion. However, in this model there is only one type of agent, investors. With only investors modeled, the welfare consequences of contagion cannot be deduced for the emerging economies. Similarly in Kyle and Xiong (2001), only investors are modeled. These authors consider wealth effects in a continuous time stochastic dynamic general equilibrium model.
with two assets and three types of investor agents—informal, informed, and noise traders. Since only investors are modeled, again the welfare implications of contagion on emerging economies cannot be deduced. Finally, Lagunoff and Schreft (2001) consider wealth links in a dynamic stochastic game-theoretic model. The authors illustrate how changes in tolerance to risk exposure after wealth losses can cause contagion. However, the change in the attitude toward risk after wealth losses in this model is not derived from agents’ preferences but it is exogenously given.

Third, a crisis can spread across countries through a portfolio recomposition channel when a crisis in one country forces investors to sell their holdings of securities in other countries.

If securities in non-crisis emerging economies exhibit some correlation with the expected return of the crisis country—due either to correlation in fundamentals or the operation of some other contagion channel—then the non-crisis country bonds will be sold with the objective of reestablishing the optimal degree of risk exposure in the investor’s portfolio. Papers that consider this link include Choueri (1999), Schinasi and Smith (1999) and Kodres and Pritsker (2002). These papers use a static partial equilibrium approach to highlight the fact that contagion might be successfully explained by standard portfolio theory.\textsuperscript{30}

**Empirical Literature**  Kaminsky and Reinhart (1998) find that the probability of crisis for any country is the highest when another country that shares investors/lenders is in crisis. No other shared characteristic with the crisis country (e.g. region, macroeconomic practices, trade links, etc.) has as large an impact on the probability of crisis. Van Rijckeghem and Weder (1999) present evidence that for the Mexican, Asian, and Russian crises spillovers through common bank lending were more significant in explaining contagion than trade links and macroeconomic similarities. Kaminsky, Lyons and Schmukler (1999) find evidence that individual share holders of open-end mutual funds followed contagion strategies in the case of the Russian crisis, and to lesser degree in the Mexican crisis: individual investors in mutual funds (not mutual fund managers) sold securities from several emerging markets when a crisis affected one of the countries. In the aftermath of the Mexican, Asian, and Russian crises, Kaminsky, Lyons, and Schmukler (2000) find that when open-end mutual funds adjusted their portfolios they considered not only the degree of fragility in fundamentals of the economies, but also factors emphasized by financial channels of

\textsuperscript{30}Specifically, the determination of asset market returns in these papers is exogenous and the pattern of correlation between returns is exogenously imposed.
contagion—openness and liquidity of the markets, as well as the level of country risk of the economies. Kaminsky and Reinhart (2000) find that during the Russian-LCTM crises, reductions in the risk exposure of investors portfolios drastically reduced the liquidity of international capital markets and increased their volatility. Hernandez and Valdes (2001) find the presence of common lenders seems to explain almost all contagion episodes during the Asian, the Russian, and the Brazilian crises. Common lenders are also highly significant in explaining contagion in stock markets.

Informational Links

Theoretical Literature The third channel of contagion, information linkages, operates in four ways.

First, incomplete information may generate contagion when the events during a crisis in one country lead investors to change their beliefs about the future performance of their investments in other countries. A crisis in one country may lead investors to believe that the economic outlook of countries with similar characteristics (but not crisis) is worse than investors previously thought. From the point of view of those investors the perceived risk in their investments in those countries increases. As consequence, even without any real change in the fundamentals of countries with similar characteristics as the crisis countries, the possibility that their economies jump from a good to a bad equilibrium emerges.

As an explanation for contagion, incomplete information suffers from a conceptual problem. Incomplete information explains contagion through the existence of multiple equilibria which allows for the possibility of jumps between equilibria without any changes in fundamentals. This characterization of the world is problematic when agents make their decisions simultaneously because there is no way for the theory to explain the severity of contagion given a shock to a first country. In other words, any pattern of comovement across countries’ capital markets is possible when investors can interpret the same information on fundamentals of emerging countries after a crisis in a first country in different ways depending on the situation. Therefore, multiple equilibria theories are equally consistent with contagion or the absence of contagion.

Drazen (1999) emphasize the incomplete information link for a currency crisis. In Drazen (1999), political contagion is considered for the case of a monetary union. The benefits of

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As investors dramatically changed their beliefs, this channel is sometimes called the wake-up call channel.

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belonging to the union depend on who else is in the union; to join the union countries must peg their currency to some stronger currency. Devaluation in one country within the union leads agents in currency markets to conclude that the benefit of belonging to the union has decreased for other countries. Therefore investors consider that a devaluation by other countries is more likely, leading investors to attack other currencies.

Second, when gathering country-specific information is costly, investors may rationally choose to act based solely on information that they can extract for free from other agents actions. This situation might lead to herding behavior. The paper that formalizes this link is Calvo and Mendoza (2000). The intuition for this kind of herding behavior is that as the number of investment opportunities grows with globalization, the variance of the world fund falls. As a result diversification is more effective regardless of whether country-specific information is collected or not.

Third, contagion might be a consequence of asymmetric information among investors. When some agents have access to a better information set than others, uninformed agents would tend to imitate informed agents. Within a stochastic general equilibrium model, Calvo (1999) formalizes this link. In Calvo’s model a crisis in one market might lead to excessive withdraws from other markets as result of idiosyncratic factors affecting informed investors, e.g. margin calls, or informed-investor-specific liquidity shocks. If informed agents are forced to liquidate positions in other markets, uninformed investors adjust their positions in those markets because of the uncertainty about the type of shock that has occurred—i.e. either informed agents have faced an idiosyncratic shock, or they have new information about those markets.

Considering asymmetric information as the source of contagion faces two empirical challenges: i) theories of contagion based on signal extraction imply a symmetric pattern of contagion that is not consistent with the empirical evidence, ii) theories of signal extraction fail in the sense that it is only possible to rationalize signal extraction in closely linked markets. Empirically contagion is not limited to these circumstances.

Fourth, informational cascades might generate contagion in a world with multiple equilibria. In the case of a crisis in one country investors would extract information from the actions of those who adjust their portfolio first. If the investors that move first believe that other economies will also succumb to crisis, these first movers will reduce their holdings of those securities. In turn, investors moving at a later date may disregard their own information about those countries when making their portfolio choice, causing capital outflows in those other countries. Investor’s sequential behavior allows multiple equilibria models to pin down one specific equilibrium, despite the existence of multiple possible outcomes. A
conceptual problem of this approach is how to justify the assumption that agents behave sequentially versus the alternative of agents behaving simultaneously.

**Empirical Literature** The wake up call channel is tested indirectly. Evidence of similarities between countries which experience contagion is regarded as support for this channel. De Gregorio and Valdes (2000), Forbes (2000), Hernandez, Mellado & Valdes (2001), and Hernandez & Valdes (2001) find that regional effects and macroeconomic similarities are one of the most important factors in explaining contagion. De Gregorio and Valdes (2000) find that for the Debt Crisis, the Mexican Crisis, and the Asian Crisis, regional effects account for most of the transmission of shocks across economies. Forbes (2000) finds that macroeconomic similarities, and country-specific characteristics can explain most of the Asian and Russian crises’ contagion. Hernandez, Mellado and Valdes (2001) find that during the two last decades contagion in capital flows can be explained mainly by regional and trade linkages. Hernandez and Valdes (2001) find that during the Asian and Brazilian crises, trade and regional effects can explain the observed contagion. As noted earlier, none of these studies controls for financial links.

Studies that control for financial links do not find support for similarities in explaining contagion. Valdes (1996) finds that for period 1986-1994 big news, such as Brady announcements, do not explain the observed contagion in creditworthiness across emerging countries. Baig and Goldfajn (1998) find that big news did not drive contagion in the Asian crisis. Kaminsky and Schmukler (1999) find that even during the Asian crisis the type of news that mattered the most in explaining comovements was neighbor countries’ news. However this channel did not account for most comovements since days of no news also exhibit large swings in asset returns. The authors interpret this as evidence of the role of investors behavior in explaining contagion.

Kaminsky and Reinhart (1998) find that between 1970 and 1995 Latin America suffered 50% more crises per country than East Asia, Europe and Middle Eastern countries, but the paper does not determine which kind of links explain this phenomena. Calvo and Reinhart (1996) find for the period 1970-1993 positive contagion in capital flows from large to small neighbors in Latin America. However this paper does not control for common shocks across economies, and includes in the measures of capital flows both private and official flows (flows from the IMF the World Bank, etc.). To sum up all of these results, the results of empirical literature that have tested for coordination channels role in contagion are not definitive in any direction.

With respect to asymmetric information the results are also mixed. Frankel and Schmuk-
ler (1996) find evidence of asymmetric information between local and international investors in emerging markets during the Mexican crisis. Using closed-end mutual fund data, Frankel and Schmukler (2000) find further evidence of differential access to information between local and international investors. However, in studying the period 1994-1998, Froot, O’Connell and Seasholes (1998) reject the hypothesis that the positive covariance of asset return and capital flows across emerging markets is associated in any way with asymmetries of information between local and foreign investors.

In summary, both information and coordination channels show mixed results in explaining contagion.