

# Default Risk and Risk Averse International Investors

Sandra Valentina Lizarazo<sup>†</sup>

First draft: March, 2005

This draft: November, 2008

## *Abstract*

This paper develops a quantitative model of debt and default for small open economies that interact with risk averse international investors. The model developed here extends the recent work on the analysis of endogenous default risk to the case in which international investors are risk averse agents with decreasing absolute risk aversion (DARA). By incorporating risk averse investors who trade with a single emerging economy, the present model offers two main improvements over the standard case of risk neutral investors: i.) the model exhibits a better fit of debt-to-output ratio and ii.) the model explains a larger proportion and volatility of the spread between sovereign bonds and riskless assets. The paper shows that if investors have DARA preferences, then the emerging economy's default risk, capital flows, bond prices and consumption are a function not only of the fundamentals of the economy—as in the case of risk neutral investors—but also of the level of financial wealth and risk aversion of the international investors. In particular, as investors become wealthier or less risk averse, the emerging economy becomes less credit constrained. As a result, the emerging economy's default risk is lower, and its bond prices and capital inflows are higher. Additionally, with risk averse investors, the risk premium in the asset prices of the sovereign countries can be decomposed into two components: a base premium that compensates the investors for the probability of default (as in the risk neutral case) and an “excess” premium that compensates them for taking the risk of default.

---

<sup>†</sup>Centro de Investigación Económica, ITAM, Mexico, D.F. Email: [slizarazo@itam.mx](mailto:slizarazo@itam.mx)  
I would like to thank to Árpád Ábrahám, Martin Uribe, Stephanie Schmitt-Grohé, and Albert ‘Pete’ Kyle for their advice. I would also like to thank Jose Wyne, and John Seater, for useful comments and suggestions. All remaining errors are my own.

# 1 Introduction

This paper extends the recent work in endogenous default risk to the case in which international investors are risk averse agents whose preferences exhibit decreasing absolute risk aversion (DARA). The current paper develops a model of debt and default for a small open economy that interacts with risk averse international investors. This model is used to account for nine stylized facts regarding emerging financial markets:

- (i) Emerging economies experience a loss of access to international capital markets and large reversals of their current account deficits in times of crises.<sup>1</sup>
- (ii) Emerging economies' domestic interest rates are counter-cyclical.<sup>2</sup>
- (iii) Default on sovereign debts occurs in equilibrium.<sup>3</sup>
- (iv) Emerging economies' credit ratings are negatively correlated with their income level and their growth rate, and positively correlated with the size of their external debt.<sup>4</sup>
- (v) Emerging economies' estimated default probabilities do not account for all of the yield spreads in their sovereign bonds.<sup>5</sup>
- (vi) The proportion of sovereign yield spreads explained by emerging economies' own fundamentals is smaller for riskier sovereign bonds than for investment grade bonds.<sup>6</sup>
- (vii) Investors' financial performance and their net foreign asset position in emerging economies are positively correlated.<sup>7</sup>
- (viii) Emerging economies' credit spreads are positively correlated with spreads of corporate junk bonds from developed countries.<sup>8</sup>

---

<sup>1</sup>The literature on "sudden-stops" has focused on explaining the dynamics of the loss of access to international capital markets that emerging economies experience during periods of crises.

<sup>2</sup>Uribe and Yue (2006) and Neumeyer and Perri (2005) focus on the counter-cyclical behavior of domestic interest rate for emerging markets.

<sup>3</sup>Kaminsky and Reinhart (1998) and Reinhart et al. (2003) document and empirically analyze default events.

<sup>4</sup>Cantor and Pecker (1996) analyzes the determinants of credit ratings.

<sup>5</sup>Westphalen (2001) and Broner et al.(2005) have considered bond spreads and the role of the probability of default in the determination of such spreads.

<sup>6</sup>See, for example, Cantor and Pecker (1996), Cunningham et al.(2001), Westphalen (2001), and Kamin and von Kleist (1999).

<sup>7</sup>See for example Goldberg (2001), Hernandez et al.(2001), FitzGerald and Krolzig (2003), and Mody and Taylor (2003).

<sup>8</sup>See, for example, FitzGerald and Krolzig (2003), Ferruci et al.(2004), and Mody and Taylor (2004).

(ix) Sovereign bond spreads across emerging economies are highly correlated.<sup>9</sup>

In the model presented here, three types of agents interact through international financial emerging markets: developed economies' agents, emerging economies' agents, and international financial intermediaries. Financial intermediaries or investors take the form of mutual funds, hedge funds, pension funds, etc. These agents invest in emerging financial markets in the name of developed economies' agents—i.e. developed economies' agents are able to invest in emerging market assets by holding shares of mutual funds, pensions or hedge funds. Since intermediaries act in tandem with developed economies' agents, these two actors will not be modeled separately. Therefore only two types agents will be explicitly modeled, the agents of the emerging economies and international investors.

It is assumed that all of the agents of the emerging economy are identical, all the international investors are identical, and that none of these agents follow mixed strategies. Under these assumptions, it is possible to focus on the representative agent of each type. For her part, the representative investor is a risk averse agent. This agent solves a dynamic portfolio problem in which she decides the optimal allocation of her portfolio between bonds of the emerging economy and riskless assets denominated as T-Bills. On the other side of the market, the representative agent of the emerging economy is also a risk averse agent who solves a dynamic optimization problem. Each period, this agent receives an stochastic endowment and chooses her consumption and savings subject to her budget constraint. The emerging economy borrows or saves by trading one-period non-contingent bonds with the representative investor. The interaction between the two parties determines the equilibrium price of the bonds in the emerging economy.

On the side of the emerging economy, there is limited liability. While the representative investor is able to commit to repay any debt that she might have, the representative agent of the emerging economy is not. In this case, the emerging economy might default on her debts. If she defaults, she is excluded from international credit markets temporarily.

Because of the enforcement problem the price of the bonds of the economy depends on the likelihood of repayment of the debt. This likelihood of repayment by the economy depends on the borrowing of the economy. Both the representative lender, and the representative agent of the economy take as given the price function of the emerging economy's non-contingent discount bonds,  $q$ .

---

<sup>9</sup>See, for example, Valdes (1996), Baig and Goldfajn (1998), Baig and Goldfajn (2000), and Forbes and Rigobon (1999).

While the investor acts as a price taker, the investors are not competitive agents. As a group, investors collude to punish any deviant borrower that defaults on a debt contract with any individual investor. Investors must also collude to punish any deviant investor that helps out a borrower who has previously defaulted.<sup>10</sup>

As laid out here, the asset market is imperfect in three different ways. First, there is a one-sided commitment problem which implies that debt contracts with the emerging economy are not enforceable. Second, markets are incomplete because the only traded assets are one period no-contingent bonds, and risk free T-Bills. Therefore the representative investor is not able to insure away the income uncertainty specific to the emerging country. Third, the market structure of the financial market is non-competitive: investors form a cartel that colludes to punish any deviant investor or borrower.

Under a non-competitive market structure, both risk averse and risk neutral investors may obtain economic profits from their lending activity. However, under this market structure, which is implicit in much of the previous sovereign debt literature, only risk averse investors are strictly better off by trading with the emerging economy. This result follows from the concavity of the periodic utility function—any risk averse agent will accept at least a small amount of any risk that is actuarially favorable. In contrast, for a risk neutral investor, once the price of the sovereign bond is adjusted by the probability of default, the investor is indifferent between trading or not trading with the emerging economy. This clarification is important because it helps to emphasize that the main departure from previous literature is the assumption of risk averse investors, and not the assumption of non-competitive investors.

By relaxing the assumption of risk neutrality, and allowing for wealth effects on the side of the international investors, the model presented here attempts to better match the facts of international financial markets during the last two decades of the 20th century. These facts are only partially explained in the existing sovereign debt literature. Under the assumption that investors are risk neutral, previous models of endogenous sovereign risk have explained stylized facts (i) through (iv).<sup>11</sup> As a result of incorporating risk averse

---

<sup>10</sup>The assumption of non-competitive investors builds on the work of Kletzer and Wright (2000), Wright (2002) and Paasche and Zin (2001). If investors were competitive agents, then once a borrower defaulted with one investor there would be no incentives for the remaining investors to exclude the defaulting borrower from financial markets. In such a case, it is not possible to support sovereign borrowing in equilibrium—it is not sufficient to merely assume reputation losses of any borrower that does not pay back.

<sup>11</sup>This literature begins with Eaton and Gersovitz (1981). More recent examples include Aguiar and Gopinath (2006), Yue(2006), Bai and Zhang (2006), Arellano (2008), Hatchondo, Martinez and Sapriza (2008) and Cuadra and Sapriza(2008).

investors with DARA preferences, the model presented here endogenously explains all of the stylized facts listed above.

The present model explains stylized facts (v) through (ix) as follows. First, international investors demand an *excess* risk premium in order to willingly take the risk of default embodied in the emerging economies' sovereign bonds (i.e. a risk averse agent would only take a risk that is actuarially favorable.). Therefore the present model is able to account for stylized fact (v): the price of the emerging economy's bonds is lower than the world price of riskless bonds adjusted by the emerging economy's default probability. This result is consistent with the findings of the empirical finance literature on sovereign bond spreads. Those findings suggest that under the assumption of risk neutral investors and competitive financial markets, the price of sovereign bonds cannot be completely explained by the estimated probabilities of default.<sup>12</sup>

Second, as risk averse agents, international investors demand a higher risk premium for higher levels of risk—above the premium predicted solely by the probability of default. With risk averse investors, the risk premium can be decomposed in two components: a base premium that compensates the investors for the probability of default and an excess premium that compensates them for taking the risk of default.<sup>13</sup> Therefore the present model is able to account for stylized fact (vi): The proportion of sovereign yield spreads explained by default probabilities is smaller for riskier sovereign bonds than for less risky bonds. This result is consistent with the empirical regularity reported in several papers: that spreads in investment grade bonds can be explained to a larger extent by emerging economies' fundamentals than spreads in speculative grade bonds.

Third, since investors' preferences exhibit DARA, these agents are able to tolerate more default risk the wealthier they are. Therefore the present model can account for stylized fact (vii): there is a positive correlation between the representative lender's wealth and the lender's investment in the emerging economy. This result is consistent with empirical findings which demonstrate a positive relation between proxies of investors wealth (like developed economies' output or stock indexes) and capital flows to emerging economies.

---

<sup>12</sup>An alternative explanation exists which does not depend on risk aversion. Sovereign bonds could be mispriced under the assumption that international investors do not take prices as given. However this assumption only explains stylized fact (v). Stylized facts (vi) through (ix) cannot be accounted for by a model in which portfolio allocations to each emerging country are independent of the wealth of the investors and the overall risk of the portfolio.

<sup>13</sup>Models with risk neutral investors only capture the base premium.

Fourth, the endogenous credit limits faced by the emerging economy become increasingly tight when the lender's risk aversion increases. This tightening occurs because a more risk averse investor demands a higher risk premium in order to accept default risk. Therefore, for any given level of risk aversion of the representative investor, the set of financial contracts available to the emerging economy is always a subset of the set of contracts available to an identical economy trading with a less risk averse lender.<sup>14</sup> This result is consistent with stylized fact (viii): whenever investors' willingness to take risk changes, there must be a change in the spreads of all risky assets. As a consequence, the spreads of emerging economies' sovereign bonds and the spreads of industrialized economies' junk bonds should exhibit some co-movement.

Fifth, under DARA preferences, investors have a higher tolerance for risk when they are wealthier. Therefore at higher levels of wealth, these agents demand a smaller risk premium than at lower levels of wealth in order to take the same amount of default risk. Furthermore, a smaller risk premium in the emerging economy's bonds increases the benefits to the economy of fulfilling its contract. Since these effects reinforce each other, the equilibrium price of sovereign bonds is an increasing function of investors' wealth levels. This result is consistent with the empirical literature on the determination of sovereign credit spreads for emerging economies,<sup>15</sup> and implies that the current model can explain stylized fact (ix): sovereign bond spreads across emerging economies are highly correlated because the equilibrium price of the emerging economy's bonds varies with the representative investor's wealth.<sup>16</sup>

In addition to the results consistent with the stylized facts above, two other results follow from the model.

First, the likelihood of observing default in equilibrium is a function not only of the emerging economy fundamentals but also of the investors' characteristics such as wealth and risk aversion. While the model does not give a definitive answer regarding the likelihood of default a priori, in the numerical simulations documented in this paper, default is more likely to be an equilibrium outcome when the investor is more risk averse.

---

<sup>14</sup>A financial contract in this context is the combination of the bond prices and quantities that the emerging economy can borrow or save.

<sup>15</sup>For example, Warther (1995), Ferruci et al.(2004), FitzGerald and Krolzig (2003), and Westphalen (2001).

<sup>16</sup>This result of the model is consistent with the literature on financial contagion. A large body of empirical literature presents evidence that financial links play a significant role in explaining simultaneous financial crises and correlated spreads across emerging economies. See, for example, Kaminsky and Reinhart (1998), Kaminsky and Reinhart (2000), Van Rijckeghem and Weder (2001), Kaminsky et al.(2001), and Hernández and Valdes (2001).

Second, this model presents a theoretical framework that can account for the non-robustness of empirical findings regarding the role of international interest rates in the determination of sovereign bond spreads and capital flows to emerging economies. In the current model changes in the world interest rate have two opposing effects on the set of financial contracts available to emerging economies:<sup>17</sup> On the one hand, an increase in the world interest rate increases the cost of borrowing for emerging economies, increasing their incentives to default and their default risk. On the other hand, an increase in this rate increases the level of wealth of non-leveraged investors; and this wealth effect would tend to increase the set of financial contracts available to emerging economies.

The assumption of DARA preferences on the side of the investors seems to be justified by the characteristics of the players in emerging financial markets. These players are both individuals and institutional investors such as banks, mutual funds, hedge funds, pension funds and insurance companies. For the case of individual investors, it is straightforward to assume that these agents are risk averse. They can be treated as the representative agent of developed economies; it is standard practice in the literature to treat these agents as risk averse. In the case of institutional investors the assumption of risk aversion is somewhat more difficult, but nevertheless quite plausible. For these investors, risk aversion may follow from two sources: regulations over the composition of their portfolio and the characteristics of the institutions' management. Regarding the first source, banks face capital adequacy ratios; mutual funds face restrictions in their access to leverage against their asset holdings; and pension funds and insurance companies face strict limits on their exposure to risk. Regarding the second source, for each class of institutional investor, managers ultimately make the portfolio allocation decisions. These managers can also be treated as representative agents of developed economies. Additionally, the remuneration—and therefore the wealth—of these agents is closely related to the performance of the portfolio that they manage. These factors suggest that portfolio choices of institutional investors will be consistent with the choices of agents whose preferences exhibit DARA.

This paper is organized as follows: section 1 is the introduction; section 2 presents the theoretical model; section 3 characterizes the equilibrium of the model; section 4 discusses the quantitative implications of the model; and section 5 concludes. Two appendixes provide proofs of propositions presented in the main text and the algorithm that solves the model.

---

<sup>17</sup>See, for example, Cline and Barnes (1997), Eichengreen and Mody (1998), Kamin and von Kleist (1999), Herrera and Perry (2000), Arora and Cerisola (2001), Reinhart et al.(2001), Goldberg (2001), Hernández et al. (2001), Kaminsky and Schmulker (2001), Bekeart, et. al. (2002), FitzGerald and Krolzig (2003), Ferruci et. al. (2004), and Mody and Taylor (2004).

## 2 THE MODEL

The model is a discrete time, infinite horizon model. There are two types of agents in the model, a representative agent small open economy, and a representative risk averse international investor. In each period, the emerging economy receives a stochastic endowment of tradable goods. The representative agent of this economy may smooth her consumption across periods by trading non-contingent discount bonds with the representative investor. For her part, the representative investor may trade assets with the emerging country or with industrialized countries. Thus the investor must choose an optimal allocation of her portfolio between the bonds of the emerging economy and bonds of the industrialized countries, denominated hereafter as T-Bills.

The market for T-bills,  $\theta^{TB}$ , will not be modeled explicitly. Since debt contracts between the representative investor and industrialized countries are assumed to be enforceable, the representative investor is a price taker in the market for T-Bills. The price of T-Bills,  $q^f$ , which is not determined endogenously in this context, is assumed to be deterministic. Therefore T-Bills are riskless assets.

Bonds of emerging economies,  $b$ , on the other hand, are risky assets because debt contracts between the representative investor and the emerging economy are not enforceable. As a consequence, there is a one sided commitment problem. While the representative investor is able to commit to honor her debt obligations with the emerging economy, the representative agent of the emerging country is not able to commit to honor her obligations with international investors. Therefore, in each period, the representative agent of the emerging economy compares the costs and benefits derived from the repayment of her obligations. The decision between repayment or default is made individually by each agent of the emerging economy. Each agent of this economy makes her decision, taking as given the decision of the other agents. However given that all agents are identical who do not follow mixed strategies, it is possible to focus attention on the problem of the representative agent.

If the economy defaults, international investors are able to collude to punish her. As a consequence of default, it is assumed that investors will collude to exclude the defaulting country for a random number of periods from the financial markets. Since all investors behave in the same exact way, it is possible to focus on the representative international investor.

Both, the representative lender and the representative agent of the economy take as given the price function of the emerging economy's non-contingent discount bonds,  $q$ .

## 2.1 International investors

There are a large but finite number of price-taking identical investors. Investors collude in order to punish any borrower that defaults on her debts or any investor that lends to a borrower who has defaulted recently, so that a defaulting country is temporarily excluded from the financial markets.<sup>18</sup>

The representative investor is a risk averse agent whose preferences over consumption are defined by a constant relative risk aversion (CRRA) periodic utility function with parameter  $\gamma^L > 0$ . The investor has perfect information regarding the income process of the emerging economy, and in each period the investor is able to observe the realizations of this endowment.

The representative investor maximizes her discounted expected lifetime utility from consumption

$$\underset{c_t^L}{Max} E_t \sum_{t=0}^{\infty} \beta_L^t v(c_t^L) \quad (1)$$

where  $c^L$  is the investor's consumption. The periodic utility of this agent is given by  $v(c^L) = \frac{(c^L)^{1-\gamma^L}}{1-\gamma^L}$ . The representative investor is endowed with some initial wealth,  $W_0$ , at time 0, and in each period, the investor receives an exogenous income  $X$ .

Because the representative investor is able to commit to honor her debt, she can borrow or lend from industrialized countries (which are not explicitly modeled here) by buying T-Bills at the deterministic risk free world price of  $q^f$ . The representative investor can also invest in non-contingent bonds of the emerging economy. These bonds have an endogenously determined stochastic price of  $q$ . In each period, the representative investor faces the budget constraint

$$W + X = c^L + dq\theta' + q^f\theta^{TB'} \quad (2)$$

where  $W$  is investors wealth at time  $t$ ,  $\theta'$  is the portfolio allocation to the emerging country and  $\theta^{TB'}$  is the investor's allocation to the riskless asset.  $d$  is an indicator variable that determines the default/repayment state of the emerging economy in the current period.  $d$

---

<sup>18</sup>As in the papers on sovereign debt literature of Kletzer and Wright (2000), and Wright(2002), no investor will deviate from this punishment as long as deviations are punished by the remaining investors. Punishment is achieved by inducing the representative agent of the emerging economy to default on her debts with the deviant investor. The colluding investors induce the emerging economy by offering her a new financial contract with slightly better terms. In this case investors will never deviate.

Empirical evidence suggest that once a country defaults, that country is excluded from the credit market for an average of 5.4 years (see Gelos, et al. (2003)).

takes the value of 1 if both the economy is not under the punishment of exclusion from financial markets as a result of a default in a previous period and the economy chooses to repay its debts, and takes a value of 0 otherwise.

It is assumed that investors cannot go short in their investments with emerging economies. Therefore whenever the emerging economy is saving, the representative international investor receives these savings and invests them completely in T-Bills. The representative investor does not use these resources to go long in T-Bills. This assumption implies that  $\theta' \geq 0$  for all  $t$ .<sup>19</sup>

The law of motion of the representative investor's wealth is given by

$$W' = d'\theta' + \theta^{TB'}. \quad (3)$$

The optimization problem that the representative investor faces can be described as one in which in each period,  $t$ , the representative international investor optimally chooses her portfolio according to her preferences in order to maximize her discounted expected lifetime utility from consumption, subject to her budget constraint, the law of motion of her wealth, and given  $W_0$ . This dynamic problem can be represented recursively by the Bellman Equation

$$V^L(s) = \max_{\theta', \theta^{TB'}} v(c^L) + E\beta_L V(s') \quad (4)$$

where  $s$  is defined as follows:

**Definition 1** *The state of the world,  $s$ , is given by the realization of the emerging economy's endowment,  $y$ , the emerging economy's asset position,  $b$ , the representative investor's asset position or wealth,  $W$ , and the variable  $d$  which states whether or not the emerging economy is in default.*

The stochastic dynamic problem for the representative investor is characterized by the first order conditions for this optimization problem:

For  $\theta^{TB'}$

$$q^f v_{c^L}(c^L) = \beta_L E \left[ v_{c^L}(c^{L'}) \right]. \quad (5)$$

---

<sup>19</sup>This assumption does not seem to be inconsistent with reality. For example, mutual funds are strictly restricted by The Investment Company Act in their ability to leverage or borrow against the value of securities in their portfolio. On the other hand, hedge funds and other types of investors face no such restrictions. Because of these regulations it seems reasonable to make the simplifying assumption that international investors are able to leverage the riskless asset,  $\theta^{TB}$ , but must have a non-negative position in the emerging economy's asset.

For  $\theta'_j$

$$d \left[ -qv_{c^L} (c^L) + \beta_L E \left[ v_{c^L} (c^{L'}) d' \right] \right] = 0. \quad (6)$$

According to Equation (5), the investor chooses an allocation to the riskless asset such that the discounted expected marginal benefit of future consumption equals the marginal cost of current consumption. Equation (6) determines the allocation of the investor's resources to the emerging country. Unless the emerging country is not in default state, i.e.  $d = 1$ , the emerging country does not belong in the investment set of the international investors. If the country is not in default state, then Equation (6) also equates the marginal cost of allocating wealth to bonds issued by the emerging country to the discounted expected marginal benefit of this investment. The benefit of this investment is realized only in those periods in which the emerging economy optimally chooses to repay its debts ( $d' = 1$ ).

For the case in which  $d = 1$ , equation (6) highlights the fact that the endogenous risk of default by the emerging economy—i.e. the case for which  $d' = 0$  for some state of the world in the next period—will reduce the representative investor's expected marginal benefit of investing in the emerging economy. Everything else equal, this result will tend to reduce the allocation of resources to the emerging economy relative to the case where the emerging economy could commit to repayment.

To understand the role that risk aversion plays in this model, it is instructive to analyze in detail the determination of the equilibrium price of the emerging economy's bonds. Define

$$Ed' = 1 - \delta$$

where  $\delta$  is the probability that the emerging economy will default in the next period.

Define  $q^{RN}$  as the equilibrium price of the emerging economy's bonds that would prevail in a world with risk neutral investors. For a risk neutral investor, the present value of one unit of a bond issued by a emerging economy that cannot commit to repay is given by

$$-q^{RN} + q^f Ed' \quad (7)$$

where  $q^f$  is used as the discount factor. This factor represents the opportunity cost for the representative investor of her investment in emerging economy bonds. Given the assumption that the investor is a price taker and that a risk neutral representative investor would make zero profits, Equation (7) implies

$$q^{RN} = q^f (1 - \delta)$$

which establishes that for the case of a representative risk neutral investor, the price of a discounted one period non-contingent bond is equal to its opportunity cost.

It is possible to manipulate equation (6) to get

$$\begin{aligned}
q &= \frac{\beta_L E \left[ v_{c^L} \left( c^{L'} \right) d' \right]}{v_{c^L} \left( c^L \right)} \\
&= \beta_L \frac{Cov \left[ v_{c^L} \left( c^{L'} \right) d' \right] + E v_{c^L} \left( c^{L'} \right) E d'}{v_{c^L} \left( c^L \right)} \\
&= \frac{\beta_L Cov \left[ v_{c^L} \left( c^{L'} \right) d' \right]}{v_{c^L} \left( c^L \right)} + q^f (1 - \delta) \\
&= \frac{\beta_L Cov \left[ v_{c^L} \left( c^{L'} \right) d' \right]}{v_{c^L} \left( c^L \right)} + q^{RN}.
\end{aligned} \tag{8}$$

Equation (8) highlights two important features of the model: i) unless the probability of default is positive, the price of the emerging economy's bonds is equal to the price of the bonds of industrialized countries; and ii) taking as given the degree of default risk (i.e. the probability of default), the price of the bonds issued by a emerging economy trading with a risk averse investor will be lower or at best equal to price of those same bonds traded with a representative risk neutral investor. This latter implication holds true because

$$Cov \left[ v_{c^L} \left( c^{L'} \right) d' \right] \leq 0.$$

When the emerging economy does not find it optimal to default at  $t + 1$  in any state of the world, then  $d' = 1$  for all states. Therefore  $Cov \left[ v_{c^L} \left( c^{L'} \right) d' \right] = 0$ . On the other hand, when at  $t + 1$  there exist states of the world in which the emerging economy would optimally choose to default, then for the states in which it is not optimal to default,  $d' = 1$ . In this case, the wealth of the representative investor at  $t + 1$  is given by

$$[W' \mid (d' = 1)] = \theta' + \theta^{TB'}$$

and the wealth of the representative investor at  $t + 1$  for the states in which the emerging economy finds it optimal to default ( $d' = 0$ ) is given by

$$[W' \mid (d' = 0)] = \theta^{TB'}$$

It is obvious that

$$[W' \mid (d' = 1)] > [W' \mid (d' = 0)].$$

Therefore it must hold that

$$\left[ c^{L'} \mid (d' = 1) \right] \geq \left[ c^{L'} \mid (d' = 0) \right]$$

and by concavity of the investor's utility function

$$\left[ v_{c^L} \left( c^{L'} \right) \mid (d' = 1) \right] \leq \left[ v_{c^L} \left( c^{L'} \right) \mid (d' = 0) \right].$$

As a consequence, for higher  $d'$ , we have lower  $v_{c^L} \left( c^{L'} \right)$ . Clearly for this case

$$Cov \left[ v_{c^L} \left( c^{L'} \right) d' \right] < 0.$$

It is important to note that the equilibrium probability of default is different in the case of a risk neutral investor,  $\delta(s, b')$ , compared to the case of a risk averse investor,  $\delta^{RN}(s, b')$ . For any given  $s$  and  $b'$ , the probability of default is an increasing function of the investor's degree of risk aversion. (This result will be studied in detail in the next section.) Therefore it is possible to say that for  $s$  and  $b'$  given, the price of the bonds issued by the emerging economy trading with a risk averse investor  $q(\delta(s, b'))$  is always lower or at best equal to price of the same bonds traded with a representative risk neutral investor  $q^{RN}(\delta^{RN}(s, b'))$ .

Compared to the case of risk neutral investors, the introduction of risk averse investors is a step forward in explaining the risk premium in the returns of bonds from emerging economies. This risk premium seems to be supported empirically since the price of emerging economies' bonds seems to be determined by much more than just the opportunity cost of the funds adjusted by the probability of default of such economies.<sup>20</sup> Risk aversion can help explain this phenomena since a risk averse investor would have to be compensated beyond the probability of default-adjusted rate of return in order to face the risk of default by an emerging economy. The higher the degree of risk aversion, the higher the bond spread.

## 2.2 The Emerging Economy

The representative agent of the emerging economy maximizes her discounted expected lifetime utility from consumption

$$\max_{\{c_t\}_{t=0}^{\infty}} E_t \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{9}$$

---

<sup>20</sup>This phenomena is discussed in Cantor and Pecker (1996) and Cunningham et al.(2001) among others.

where  $0 < \beta < 1$  is the discount factor and  $c$  is the emerging economy's consumption at time  $t$ . The emerging economy's periodic utility takes the functional form

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

where  $\gamma > 0$  is the coefficient of relative risk aversion.

In each period, the economy receives a stochastic stream of consumption goods,  $y$ . This endowment is non-storable; realizations of the endowment are assumed to have a compact support; and the endowment follows a Markov process drawn from probability space  $(y, Y(y))$  with a transition function  $f(y' | y)$ .

In each period, based on the stochastic endowment,  $y$ , the economy decides how much to consume,  $c$ . The economy can consume  $c > y$  by trading one period non-contingent discount bonds  $b'$  at a price,  $q$ , with international investors.

As a consequence of commitment problems, the price of the emerging economy's bond might be different depending on whether the economy is saving or borrowing. If  $b' > 0$ , the country is saving, and because the international investor is able to commit, there is no risk of default on such a bond. In this case, the emerging economy's bond is identical to the bonds issued by industrialized markets; therefore, because the representative investor is a price taker, in equilibrium the bond price of a emerging economy with no default risk is the same as the bond price of industrialized countries. Consequently, the price of a bond with a positive face value is equal to the price of a T-Bill, so  $q = q^f$ .

If  $b' = 0$ , the emerging economy is not borrowing and there is no risk of default because it is not optimal for the emerging economy to declare default on a debt of size 0. If the economy were to declare default in this circumstance, there would be no change in the present pattern of consumption, but a reduction in the opportunities of consumption smoothing in the future.

If  $b' < 0$ , the emerging country is borrowing. In this case, because emerging economies cannot bind themselves to honor their debts, the emerging country might default next period. At one extreme, there might be values of  $b' < 0$  for some given state of the world,  $s$ , such that the representative agent of the economy never finds it optimal to default. In this case the bonds issued by the emerging economy do not involve any default risk, and therefore  $q = q^f$ . At the other extreme, for the same state of the world,  $s$ , there might be values of  $b' < 0$  such that once the debt is due the economy would not choose to repay in any state of the world next period,  $s'$ . In this case  $q = 0$ . In the intermediate case, for the same state of the world,  $s$ , some other values of  $b' < 0$  might imply that the emerging

economy will find it optimal to default on her debts in some states of the world next period  $s'$ . In this case, in order to induce international investors to buy the emerging economy's bonds, the price of such bonds needs to be lower than the price of a T-Bill,  $q < q^f$ .

Based on this logic, the price of the emerging economy's bonds is a function not only of the state of the world,  $s$ , but also of  $b'$ .

The resource constraint of the emerging economy is given by

$$c = y + d(b - qb'), \quad (10)$$

where  $d$ , which has been defined in the investor's section, describes the state of the economy with respect to participation in international financial markets. If  $d = 1$ , the economy is not in a state of default. If  $d = 0$  the emerging economy is in a state of default (either because she has defaulted on her debts in a previous period and has not regained access to financial markets or because she is defaulting on her debts in the current period) and this country is currently in financial autarky. Once a country defaults (even if the default is partial), that country is excluded from access to the credit market, and that country remains in a state of default for a random number of periods. During the periods of exclusion of financial markets the country is not able to smooth its consumption, and it is limited to consume its stochastic endowment.

Under this framework, the optimization problem of the emerging country can be represented recursively by the following Bellman equation

$$V(s) = \max \{V^C(s), V^D(s)\} \quad (11)$$

and

$$\begin{aligned} V^C(s) &= \max_{c, b'} u(c) + \beta EV(s' | s) \\ \text{s.t.} \quad c &= y + b - qb' \\ \& \quad b' &\geq \underline{b} \end{aligned} \quad (12)$$

where  $V^C(s)$  is the value to the economy of not defaulting and  $V^D(s)$  is the value of defaulting in the current period.

**Definition 2** *The value for the emerging economy of default is given by*

$$V^D(s) = u(y) + \beta E[\tau V^C(s' | s) + (1 - \tau)V^D(s' | s)].$$

*where  $\tau$  is the exogenous probability that the emerging economy would re-enter credit markets in the current period given that this economy has defaulted in her debts in a previous period.*

For the emerging country the decision of default/repayment depends on the comparison between the value of continuing in the credit contract,  $V^C(S)$ , versus the value of opting of financial autarky,  $V^D(S)$ . The decision of current default/repayment takes the functional form

$$d = \begin{cases} 1 & \text{if } V^C(s) > V^D(s) \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

Equation (12) corresponds to the “natural” debt limit discussed in Aiyagari (1994), which prevents the representative agent of the emerging economy from running ponzi games. In the current model, this constraint would not be binding. Instead, a tighter credit limit is determined endogenously in the model.

The stochastic dynamic problem for the emerging economy is characterized by the Euler equation (conditional on not defaulting in the current period):

$$u_c(c(s)) q \left( 1 + \frac{\partial q(s; b')}{\partial b'(s)} \frac{b'(s)}{q(s; b')} \right) = \beta E [u_c(c'(s')) d'(s')] \quad (14)$$

and equations (10) and (13).

The Euler equation (14) equates the marginal benefit of one unit of current consumption to the discounted expected marginal cost of giving up one unit of future consumption. Because of the commitment problem, this cost is experienced only in those states in which the emerging economy optimally chooses to repay its debt, i.e. only on those states in which  $d' = 1$ .

Equations (14) and (6) make clear that for the case of an economy that cannot commit to repayment, when there exist levels of  $b'$  in which the emerging economy finds it optimal to default in some states of the world, then the price of bonds depends not only on the emerging economy’s fundamentals, but on the representative investor’s level of wealth and risk aversion. This case is very different from the case of an identical small open economy that faces risk neutral investors in international financial markets. As can be seen in other models of the sovereign debt literature when investors are risk neutral, the price of bonds of the economy depends only on the economy’s own fundamentals and characteristics.

### 3 Characterization of the Equilibrium

The recursive equilibrium in this model is given by prices,  $q^f$  and  $q$ , and quantities,  $c, b', d, \theta'$ , and  $\theta^{TB'}$  which solve the optimization problems of the emerging economy and the representative investor.

The equilibrium prices and quantities must also clear asset markets:

$$b' = -\theta' \quad \text{if } b' < 0 \quad (15a)$$

$$0 = -\theta' \quad \text{if } b' \geq 0. \quad (15b)$$

Equations (15a) and (15b) imply that in equilibrium the emerging economy and the representative investor agree on a financial contract,  $b'$  and  $q$ , that is optimal for both agents.

**Definition 3** *For a given level of wealth,  $W$ , the default set  $D(b | W)$  consists of the equilibrium set of  $y$  for which default is optimal when the emerging economy's asset holdings are  $b$ :*

$$D(b | W) = \left\{ y \in Y : V^C(s) \leq V^A(y) \right\}. \quad (16)$$

Equilibrium default sets,  $D(b' | W'(s))$ , are related to equilibrium default probabilities,  $\delta(b', y' | s)$ , by the equation

$$\delta(b', y' | s) = 1 - \int_{D(b' | W'(s))} f(y' | y) dy' \quad (17)$$

If the default set is empty for  $b'$ , then for all realizations of the economy's endowment,  $d' = 1$  and the equilibrium default probability  $\delta(b', y' | s)$  is equal to 0. In this case, it is not optimal for the economy to default in the next period for any realization of its endowment, and  $Cov[v_{cL}(c^{L'}) d'] = 0$  and  $q = q^f$ . On the other hand, if the default set includes the entire support for the endowment realizations, i.e.  $D(b' | W'(s)) = Y$ , then  $d' = 0$  for all realizations of the economy's endowment. As a consequence, the equilibrium default probability  $\delta(b', y' | s)$  is equal to 1, and  $Cov[v_{cL}(c^{L'}) d'] = 0$ , so  $q = 0$ .

Otherwise, when the default set is not empty but does not include the whole support for the endowment realizations,  $0 < \delta(b', y' | s) < 1$ . In this case, which was analyzed in the section describing the investors' optimization problem,  $Cov[v_{cL}(c^{L'}) d'] > 0$ , so  $q < q^f$ .

### 3.1 Characterization of Default Sets

The characterization of default sets is the characterization of incentives to default and therefore the characterization of endogenous default risk. In this model, default risk is a function of both the emerging economy's fundamentals—the economy's endowment process and its asset position—and the characteristics of the international investor—the investor's risk aversion and wealth.

**Endogenous Credit Constraints and Maximum Safe Level of Debt** In order to continue with the characterization of the default sets it is necessary to define two concepts, the *endogenous credit constraint* and the *maximum safe level of debt*. The *endogenous credit constraint* is the maximum level of assets,  $\underline{b}(W)$ , that is low enough such that no matter what the realization of the endowment, default is the optimal choice and  $D(\underline{b}(W) | W) = Y$ . On the other hand, the *maximum safe level of debt* is the minimum level of assets  $\bar{b}(W)$  for which staying in the contract is the optimal choice for all realizations of the endowment. In this case,  $D(\bar{b}(W) | W) = \emptyset$ . Finally, because the value of the credit contract is monotonically decreasing in  $b$ , it is obvious that

$$\underline{b}(W) \leq \bar{b}(W) \leq 0 \quad .$$

**Proposition 1** *For any state of the world,  $s$ , the endogenous credit constraint,  $\underline{b}(W)$ , and the maximum safe level of debt,  $\bar{b}(W)$ , are single-valued functions.*

**Proof.** To define these concepts, note that the stochastic process for the endowments has a compact support. Also note that, conditional on  $W$ , the value of the credit contract is monotonically decreasing in  $b$ . Monotonicity of the credit contract and compactness of the endowment support are sufficient conditions to guarantee that given the state of the world, these critical values (i.e. endogenous credit constraint and maximum safe level of debt) are single-valued functions. ■

From the previous discussion it is clear that given some current level of investors' wealth, any investment in the emerging economy's bonds in excess of  $\underline{b}(W)$  would imply a probability of default equal to 1. These investments will have a price of 0. On the other hand, all investments in the emerging economy's bond of an amount lower than  $\bar{b}(W)$  imply a zero probability of default. These investments will have a price of  $q^f$ .

**Default Sets and Risk Aversion of International Investors** The degree of investors' risk aversion is an important determinant of access of emerging economies to credit markets, and of the risk of default of the economy. In this model, the more risk averse are international investors, the higher is the default risk and the tighter is the endogenous credit constraint faced by all emerging economies.

**Proposition 2** *For any state of the world,  $s$ , as the risk aversion of the international investor increases, the emerging economy's incentives to default increase.*

**Proof.** See Appendix. ■

The economic intuition behind this result is straightforward for the case in which the exclusion of credit markets following a default is permanent. In this case, for the emerging economy, while the value of autarky is not a function of the investor’s risk aversion, the value of maintaining access to credit markets is decreasing in the lender’s degree of risk aversion. In order to induce a very risk averse investor to hold sovereign bonds, the emerging economy has to forgo much more current consumption—i.e. has to accept a very low price for her bonds. Other things equal, with lower bond prices, incentives to default are stronger. Therefore for any given state of the world,  $s$ , the degree of risk in the economy is increasing in the degree of risk aversion of international investors.

When the exclusion from credit markets following default is not permanent, both the value of financial autarky and the value of maintaining access to credit markets are functions of the investor’s risk aversion. The value of autarky in the current period includes the value of maintaining access to the credit markets in future periods once the economy is admitted back to the credit markets; therefore is also decreasing in the investor’s degree of risk aversion. However the value of maintaining access to credit markets in future periods is weighted by the probability of coming back (which is lower than 1) and the discount rate (which is also lower than 1). As a consequence, the value of autarky in the current period is less responsive to the risk aversion of international investors than the value of maintaining access to credit markets in the current period. Therefore, the degree of risk in the economy is increasing in the degree of risk aversion of international investors, as in the case of permanent punishment after a default.

As the degree of risk in the economy changes, so too will the capital flows to the economy: For  $\gamma_L^1 < \gamma_L^2$ , Proposition 2 implies that

$$D(b | W; \gamma_L^1) \subseteq D(b | W; \gamma_L^2).$$

Therefore, it must hold that

$$\begin{aligned} \underline{b}(W; \gamma_L^2) &\geq \underline{b}(W; \gamma_L^1). \\ \overline{b}(W; \gamma_L^2) &\geq \overline{b}(W; \gamma_L^1). \end{aligned}$$

This equation shows that endogenous credit constraints  $\underline{b}(W)$  for the emerging economy are tighter the more risk averse are international investors—some contracts that are feasible under less risk adverse investors are not feasible under more risk averse investors.

The result in Proposition 2 is consistent with empirical findings which characterize the role of investor's risk aversion in the determination of country risk and sovereign yield.<sup>21</sup>

**Default Sets and Investor's Wealth** In the present model, the economic performance of the emerging economy cannot be explained by the fundamentals of the emerging economy alone, i.e. by the economy's asset position and stochastic process of the endowment. The investor's wealth also affects the emerging economy's performance. This result is formalized in Proposition 3.

**Proposition 3** *Default sets are shrinking in assets of the representative investor. For all  $W_1 < W_2$ , if default is optimal for  $b$  in some states  $y$ , given  $W_2$ , then default will be optimal for  $b$  for the same states  $y$ , given  $W_1$ . Therefore  $D(b | W_2) \subseteq D(b | W_1)$*

**Proof.** See Appendix. ■

The intuition for Proposition 3 is simple: given some default risk, it is less costly (in terms of current utility) for the investor to invest in the emerging economy when she is wealthy than when she is poor. So keeping constant the degree of risk that the investor faces, any investment that she is willing to undertake when she is poor she will also be willing to undertake when she is rich. Intuitively, financial contracts available to the representative agent of the emerging economy when the investor is relatively rich have to be at least as good as the feasible contracts to which the economy has access when the investor is relatively poor. Additionally, the previous effect implies that the emerging economy faces stronger incentives to default when the wealth of the investor is relatively low. Therefore default risk is decreasing in the wealth of the investor. These two effects amplify and reinforce each other.

---

<sup>21</sup>Much empirical evidence supports Proposition 2: Using the spread between the yield of three month T-bills and the US federal funds rate as a proxy for market turbulence, Arora and Cerisola (2001) find that heightened macroeconomic uncertainty in the US, has a positive significant effect on sovereign credit spreads for emerging markets. Using high-low yield spreads on US corporate bonds as a proxy for risk aversion of US investors, Ferruci et al.(2004) and FitzGerald and Krolzig (2003) find that sovereign bond spreads increase when the risk aversion of US investors increases. Similarly, Cunningham et al. (2001), Westphalen (2001), and Kamin and von Kleist (1999) find evidence that the risk premium in sovereign bonds increases more than proportionally when default risk increases. Finally, Mody and Taylor (2004), Ferruci et al.(2004), and FitzGerald and Krolzig (2003) find that risk aversion of US investors is an important determinant of capital flows to emerging economies: a higher US high-low yield spread—interpreted as a reduction in investor risk appetite—results in a reduced supply of capital to emerging economies.

Proposition 3 implies that for  $W_1 < W_2$  it must hold

$$\begin{aligned}\underline{b}(W_1) &\geq \underline{b}(W_2) \\ \overline{b}(W_1) &\geq \overline{b}(W_2)\end{aligned}$$

and therefore the endogenous credit limit that the emerging economy faces is tighter for lower levels of wealth of the investor ( $\underline{b}(W_1) \geq \underline{b}(W_2)$ ).

This result is a consequence of the fact that for investors, the marginal cost of investing in sovereign bonds in terms of current consumption is decreasing in investors' wealth. Given that these agents are risk averse with decreasing absolute risk aversion, investing in the sovereign bonds when their wealth is low is too costly; so when the wealth of the investor falls, the resources available to the emerging economy become scarce, reducing the value for the emerging economy of participating in credit markets. In turn, because the sovereign country has increasing incentives to default, some loans or portfolio investments that are feasible when the investor is wealthy cannot be an equilibrium outcome when the investor is poor.

Findings of several empirical papers on the literature regarding the determinants of capital flows and sovereign bonds spreads of emerging economies are consistent with the results in Proposition 3. See, for example, Warther (1995), Westphalen (2001), FitzGerald and Krolzig (2003), Mody and Taylor (2004), and Ferruci et al.(2004).

The results in Proposition 3 are also consistent with the evidence regarding financial contagion across countries who share investors. See for example Kaminsky and Reinhart (1998), Kaminsky and Reinhart (2000), Hernandez and Valdes (2001) and Van Rijckeghem and Weder (2001).<sup>22</sup>

**Default Sets and the Asset Position of the Emerging Economy** In the model, a highly indebted economy is more likely to default than an economy with lower debt. And as in models of the same type where investors are risk neutral, default sets are shrinking in assets.

---

<sup>22</sup>For the period 1984 to 1993, Warther (1995) finds that an inflow to corporate bond funds of around 1% of the mutual fund's assets results in a permanent increase of 2.1% in those bond prices (i.e. reduces the cost of borrowing for those issuing those bonds). Using world and U.S. equity indexes respectively as proxies for the business climate (an increase in these indexes is associated with a better business climate), Westphalen (2001) and Ferruci et al. (2004) find a negative relation between economic expansion in the investors' countries and sovereign yield spreads of emerging economies. FitzGerald and Krolzig (2003) find a positive and significant relationship between US output and capital inflows to emerging economies. Finally, Mody and Taylor (2003) find that a higher growth in industrial production in the US has a positive effect on the supply of capital to emerging economies.

**Proposition 4** *Default sets are shrinking in assets of the emerging economy. For all  $b_1 < b_2$ , if default is optimal for  $b_2$  in some states  $y$ , given  $W$ , then default will be optimal for  $b_1$  for the same states  $y$ , given  $W$ . Therefore  $D(b_2 | W) \subseteq D(b_1 | W)$ .*

This result is analogous to the result in Arellano (2008), and closely related to the results in Eaton and Gersovitz (1981) and Chatterjee, et al. (2002). The main difference in the present paper is that the result is conditioned on the level of wealth of the representative investor.

The economic intuition is simple for the case in which the punishment for defaulting is permanent exclusion from credit markets and is as follows. While the value for the economy of fulfilling the contract is increasing in  $b$ , the outside value of the economy is not—the value of financial autarky does not depend on  $b$ . Therefore as the indebtedness of the economy increases, the value of the contract decreases, while the value of default remains unchanged. As a consequence, starting from an asset position  $b$  in which default is the optimal choice, it is clear that if the assets shrink, the value of the contract also falls. As the value of the contract falls, default will continue to be the optimal choice.

The quantitative analysis of the model shows that this result follows through to the more general case in which following a default the economy is excluded from credit markets only temporarily.

**Default Sets and Endowment Realization** Default sets also depend on the realization of income. As in Arellano (2008), it is possible to show analytically that for the case of permanent exclusion of the emerging economy from credit markets after defaulting, if the endowment process is *i.i.d.* for given  $W$ , then default incentives are stronger for lower levels of income.

The numerical solution of the present model extends this result to the more general case in which the exclusion of the emerging economy from credit markets after defaulting is not permanent and the stochastic process of her endowments follows a Markov chain with persistence.

**Proposition 5** *If the endowment process is *i.i.d.*, default incentives are stronger the lower the endowment. For all  $y_1 < y_2$ , if  $y_2 \in D(b | W)$  then  $y_1 \in D(b | W)$ .*

The intuition for this result follows Arellano (2008). Again, the main difference is that in the present context, the result is conditioned on the level of wealth of the investors.

The logic behind this results follows from the fact that default is only optimal if under all feasible financial contracts the emerging economy experiences capital outflows.<sup>23</sup> In the case of a recession, capital outflows are extremely costly in terms of the welfare of a risk averse agent (because the concavity of the periodic utility); therefore at sufficiently low levels of the endowment realization, the credit market becomes a less effective tool for consumption smoothing than default.

This result is also consistent with the empirical literature on the determination of credit ratings and sovereign yields. In this literature, sovereign yield spreads increase when the economy’s fundamentals deteriorate, mainly when output falls.

Additionally, this result implies that because default risk is counter-cyclical, domestic interest rates are also counter-cyclical. Counter-cyclicity is consistent with the stylized facts of financial emerging markets (see Neumeier and Perri (2005), and Uribe and Yue (2006)).

### 3.2 Default as an equilibrium outcome of the model and Investors characteristics

In the current model, default can be an equilibrium outcome if the emerging economy finds it optimal to choose  $b'$  such that  $D(b' | W'(s)) \neq \emptyset$ . In other words, to observe default at equilibrium, it must hold that beginning from an asset position  $b$ , such that  $D(b | W) = \emptyset$ , there exists a sequence of endowment shocks such that this economy ends up borrowing  $b'$  and  $D(b' | W'(s)) \neq \emptyset$ . As in the case in which international investors are risk neutral studied in Arellano (2008), this outcome is possible only if the equilibrium price function does not decrease “too fast” when assets decrease. Default is a possible outcome at equilibrium only if by increasing its borrowing to levels for which there is default risk, the emerging economy is able to increase current period capital inflows  $b - q(s, b')b'$ . In this case, by borrowing more and more, the economy achieves a higher level of consumption

---

<sup>23</sup>The current model shares one feature with models of the same kind in which investors are risk neutral: the emerging economy only defaults when it is facing capital outflows. In this case,  $d(s) = 0$  implies that for all the financial contracts available to the economy,  $b - q(s; b'(s))b'(s) < 0$ . Intuitively, whenever the emerging economy decides to default, the value of default must be at least as good as the value of the optimal financial contract available to this country ( $V^C(s) \leq V^D(s)$ ). However if any available financial contracts allows for capital inflows to the emerging economy, then by choosing that contract the economy not only can consume more in the current period than under financial autarky ( $c > y$ ), but in the next period the economy is guaranteed at least the same level of satisfaction as under autarky (because the economy has the option of defaulting in the next period). Therefore for any state of the world  $s$ , whenever there are financial contracts  $\{q(s; b'(s)), b'(s)\}$  such that  $b - q(s; b'(s))b'(s) > 0$ , default is not an optimal decision.

even though the economy has to accept a lower price for its bonds in order to compensate the investors for taking the risk of default.

**Proposition 6** *Given  $\bar{b}(W'(s); f(y'|y), \gamma^L)$ , default at equilibrium is a possible outcome of the time series of the model if for  $b' = \bar{b}(W'(s); f(y'|y), \gamma^L)$*

$$\frac{\partial c}{\partial \bar{b}(\cdot)} = -\frac{\partial q(\cdot) \bar{b}(\cdot)}{\partial \bar{b}(\cdot)} < 0.$$

*In other words, default can be an equilibrium outcome if for  $b' = \bar{b}(W'(s); f(y'|y), \gamma^L)$ , it holds  $\frac{\partial q(\cdot) \bar{b}(\cdot)}{\partial \bar{b}(\cdot)} > 0$ , so that by increasing its borrowing, the emerging economy is able to increase its consumption.*

The sign of this derivative (i.e.,  $\frac{\partial q(\cdot) \bar{b}(\cdot)}{\partial \bar{b}(\cdot)}$ ) is ambiguous depending on both the emerging economy's fundamentals and investors' characteristics:

$$\begin{aligned} \frac{\partial q(\cdot) \bar{b}(\cdot)}{\partial \bar{b}(\cdot)} &= \overbrace{q(s, \bar{b}(W'(s)); f(y'|y), \gamma^L)}^{>0} \\ &+ \underbrace{\bar{b}(W'(s); f(y'|y), \gamma^L)}_{\leq 0} \underbrace{\frac{\partial q(s, \bar{b}(W'(s)); f(y'|y), \gamma^L)}{\partial \bar{b}(W'(s); f(y'|y), \gamma^L)}}_{\geq 0} \leq 0 \end{aligned} \quad (18)$$

given

$$\frac{\partial q(s, \bar{b}(W'(s)); f(y'|y), \gamma^L)}{\partial \bar{b}(W'(s); f(y'|y), \gamma^L)} = \beta_L E \frac{v_{c_L}(c'_L)}{v_{c_L}(c_L)} \left[ \left( \frac{\gamma_L}{c'_L} + \frac{\gamma_L}{c_L} q \right) d' + \frac{\partial d'}{\partial \bar{b}} \right] \quad (18a)$$

$$q(s, \bar{b}(W'(s)); f(y'|y), \gamma^L) = q^f. \quad (18b)$$

Roughly speaking, the smaller is the equilibrium maximum safe level of borrowing,  $\bar{b}(W'(s); f(y'|y), \gamma^L)$ , the higher is the chance that this derivative turns out to be positive. Intuitively, because investors must be compensated in order to induce them to take some default risk, this risk imposes an additional cost of borrowing for the emerging economy. For the borrower, the cost of borrowing beyond the maximum safe level must be paid over the total amount of resources borrowed, and not only over the marginal amount of borrowing. Therefore, the larger is the base over which this additional cost of borrowing has to be paid—i.e. the larger is the maximum safe level of borrowing—the higher is the cost of default risk and the lower is the likelihood that the economy would ever choose to borrow beyond the safe level of debt.

**Role of  $W$  in the determination of the sign of  $\frac{\partial q(\cdot)\bar{b}(\cdot)}{\partial \bar{b}(\cdot)}$ .** Other things given, the sign of  $\frac{\partial q(\cdot)\bar{b}(\cdot)}{\partial \bar{b}(\cdot)}$  is more likely to be positive when the level of investors' wealth is lower.

First, because of Proposition 3,  $\frac{\partial \bar{b}(\cdot)}{\partial W(\cdot)} < 0$ , i.e. a higher level of investor's wealth allows the emerging economy to borrow more. This effect implies that when investors are wealthier, other things equal, default risk imposes a larger additional cost of borrowing beyond the safe level of debt (i.e.  $\bar{b}(\cdot) \frac{\partial q(\cdot)}{\partial \bar{b}(\cdot)}$  is larger). In this case, any change in the price of the sovereign bonds will be felt over a larger base of borrowing. As a result, for the emerging economy there is potentially less to gain from accepting a lower price for these bonds in order to further increase borrowing. This effect makes it more difficult for the economy to increase consumption by risking default. Consequently, this effect implies that it should be easier to observe default as an equilibrium outcome when international investors are relatively constrained financially compared to when investors are relatively solvent.

Second, a higher level of investors' wealth reduces the absolute risk aversion of these agents,  $\left(\frac{\gamma_L}{c_L} \& \frac{\gamma_L}{c_L}\right)$ . As a consequence, because the investors demand a relatively small *excess* risk premium, sovereign bond prices change "more slowly"—that is,  $\frac{\partial q(\cdot)}{\partial \bar{b}(\cdot)}$  is smaller in absolute terms.

These two effects go in opposite directions. Therefore, is not possible to establish analytically how the equilibrium default probability of the model responds to changes in the wealth level of the investors. The numerical simulations of the model performed here suggest that this effect can go either way.

**Role of  $\gamma_L$  in the determination of the sign of  $\frac{\partial q(\cdot)\bar{b}(\cdot)}{\partial \bar{b}(\cdot)}$ .** It is also more likely that the sign of  $\frac{\partial q(\cdot)\bar{b}(\cdot)}{\partial \bar{b}(\cdot)}$  is positive for higher levels of investors' risk aversion.

First, Proposition 2 establishes that  $\frac{\partial \bar{b}(\cdot)}{\partial \gamma_L} > 0$ , i.e. the more risk averse investors are, the less the economy is able to borrow and the lower is the maximum safe level of borrowing for any given state of the world. Therefore, other things equal, if the investor is very risk averse, the cost of a change in the price of the bonds is felt over a smaller borrowing base. In this case, there is potentially more to gain from accepting a lower price for these bonds in order to further increase borrowing. Therefore this effect makes default a more likely outcome of the model.

Second, larger risk aversion of the investor also implies a larger response of  $q(\cdot)$  to changes in the borrowing level. Other things equal, the more risk averse is the investor, the larger is the excess risk premium that she demands in order to take default risk.

Table 1: Business Cycle for Argentina and International Investors

	std(x)	corr(x,output)	corr(x,output usa)	corr(x,spread)
Interest Rate Spread	5.42	-0.54	-0.10	
Trade Balance	2.17	-0.42	-0.14	0.71
Consumption	6.33			
Output	5.93		0.09	
Consumption USA	0.80	0.03	0.80	-0.08
Output USA	0.98	0.09		
SP500	8.87	0.16	0.19	-0.33
Dow	9.69	0.13	0.03	-0.21
Vix	23.09	-0.19	-0.22	0.07

Again these two effects go in opposite directions. Therefore, is not possible to establish analytically how the equilibrium default probability of the model responds to changes in the investor's risk aversion. However, the numerical simulations of the model performed here suggest that this effect makes default at equilibrium more likely whenever investor's risk aversion is higher.

## 4 Quantitative Analysis

Following the recent literature on endogenous sovereign default risk, the model in this paper is used to study the case of Argentina and its default at the end of 2001. The model is solved numerically at a quarterly frequency and its parameters are chosen to replicate some features of the argentinean economy and the international investors in emerging economies for the period 1983:Q1-2001:Q4.

Table 1 describes the relevant business cycle features for the period under study. For the Argentinean output, consumption and trade balance, and for the U.S. output and consumption the source of the data is the IFS; for the yield of 3-months U.S. Treasury Bills the source is the Federal Reserve Board; for the SP500 index and the Dow-Jones Industrial Average index the source is Bloomberg; for the CBOE Volatility Index (VIX) the source is the CBOE; finally for the interest rate of Argentina the source is Neumeyer and Perri (2005). The data for the business cycle statistics includes the period 1983:Q1-2001:Q4 for all series except for Argentina's private consumption and for VIX. For Argentina's private consumption data is only available from 1993:Q1 on and for the VIX data is available only from 1990:Q1 on. Therefore for these two variables the business cycle statistics corresponds to the period from the initial moment in which each of them is available to first quarter of 2008. Output and consumption for Argentina and the U.S., and the sp500, the Dow-Jones

Table 2: Parameter Values

Parameter	Value
Emerging Economy's Risk Aversion $\gamma$	2
Representative investor's Risk Aversion $\gamma^L$	0.5, 1, 2
Emerging Economy's Mean Income $E[y]$	1
Emerging Economy's Discount Factor $\beta$	0.95
Representative Investor's Discount Factor $\beta^L$	0.9875
Risk Free Interest Rate $r^f = \frac{1}{q^f}$	0.01
Representative investor's Income $X$	0.0107
Probability of re-entry $\tau$	0.1, 0.2, 0.5
Proportional Output cost of defaulting	0.02
Critical level of output for asymmetrical output cost	$\hat{y} = 0.969E(y)$
Transitory Shocks	
Std. Dev. Emerging Economy's Income $std[y]$	0.0342
Autocorr. Emerging Economy's Income Process	0.7294
Permanent Shocks	
Growth Rate. Emerging Economy's Income $g_y$	0.006
Std. Dev. Emerging Economy's Income $std[y]$	0.03
Autocorr. Emerging Economy's Income Process	0.17

and the VIX indexes are in logs. The Argentinean trade balance is reported as a percentage of the output. The interest spread is defined as the difference between the Argentinean interest rate and the yield of a 3 month U.S. T-Bill. All data are filtered with the H-P filter.

As has been documented in the previous literature on the subject of default risk, interest rate spreads are negatively correlated with the Argentinean output and consumption. The model in this paper matches this correlation; additionally, if we consider that investor's wealth can be proxy by either the U.S. output, the Dow-Jones index or the SP500 index and that investor's absolute risk aversion can be proxy by the VIX index, the model matches the observed correlation between spreads and international investors wealth and investors absolute risk aversion. <sup>24</sup>.

## 4.1 Calibration

Table 2 gives the parameters which are considered in the numerical analysis of the model. The coefficient of risk aversion is 2, a standard value considered in the business cycle litera-

---

<sup>24</sup>The Dow-Jones index is a price-weighted index of 30 blue-chip stocks from U.S. firms that are generally leaders in the industry. The SP500 index is a capitalization-weighted index of 500 stocks that represent all industries that is designed to measure performance of the broad economy. The VIX index is a measure of the implied volatility of the SP500, and a high value corresponds to a more volatile market.

ture. The representative investor's coefficient of risk aversion is set at three different values 0.5, 1, and 2 to determine the impact of changing investors' risk aversion.

The mean income of the emerging economy is normalized to 1. The representative investor, on the other hand, receives a deterministic income of 0.0107, (or 1% of the emerging economy's mean income) in each period.<sup>25</sup> This parameter is chosen using information about the relative importance of fees and trading costs that investors like mutual funds charge to individual investors that diversify their portfolios through them.

The free interest rate is set to 1%, to match the quarterly US interest rate.

The representative investor's discount factor is set to 0.9875 which is in the range commonly used in business cycle studies of industrialized countries.

The probability of re-entry to credit markets after defaulting is set at three alternative values 0.1, 0.2, and 0.5 that have been considered in the previous literature; these values are consistent with the empirical evidence regarding the exclusion from credit markets of defaulting countries (see Gelos et al. (2002)).

Two alternative scenarios are considered for the output loss that follows a default: first, a proportional output loss of 2% that is identical to the one assumed in Aguiar and Gopinath (2006); and second, an asymmetrical output loss for output values higher than a critical value  $\hat{y} = 0.969E(y)$  that is assumed in Arellano (2008).

Regarding the income process for the emerging economy's endowment two alternative scenarios are considered: first, following Arellano (2008) only transitory shocks are considered. Under this assumption the standard deviation of the income process for the emerging economy's endowment is set to 3.42% and the auto-correlation of the process is assumed to be 0.7294. Second, following Aguiar and Gopinath (2006), shocks to the income growth trend are considered. In this case the mean quarterly growth rate is set to 0.6%, the standard deviation of the income process is set to 3%, and the auto-correlation of the income process is assumed to be 0.17.

---

<sup>25</sup>The parameter  $X$  is the main determinant of the natural credit limit faced by international investors, i.e. the no-ponzi condition. The larger this parameter is, the looser the credit limit, and the wealthier are the investors; consequently when this parameter is large, the impact of changes in wealth over the optimal investors' portfolio is relatively small.

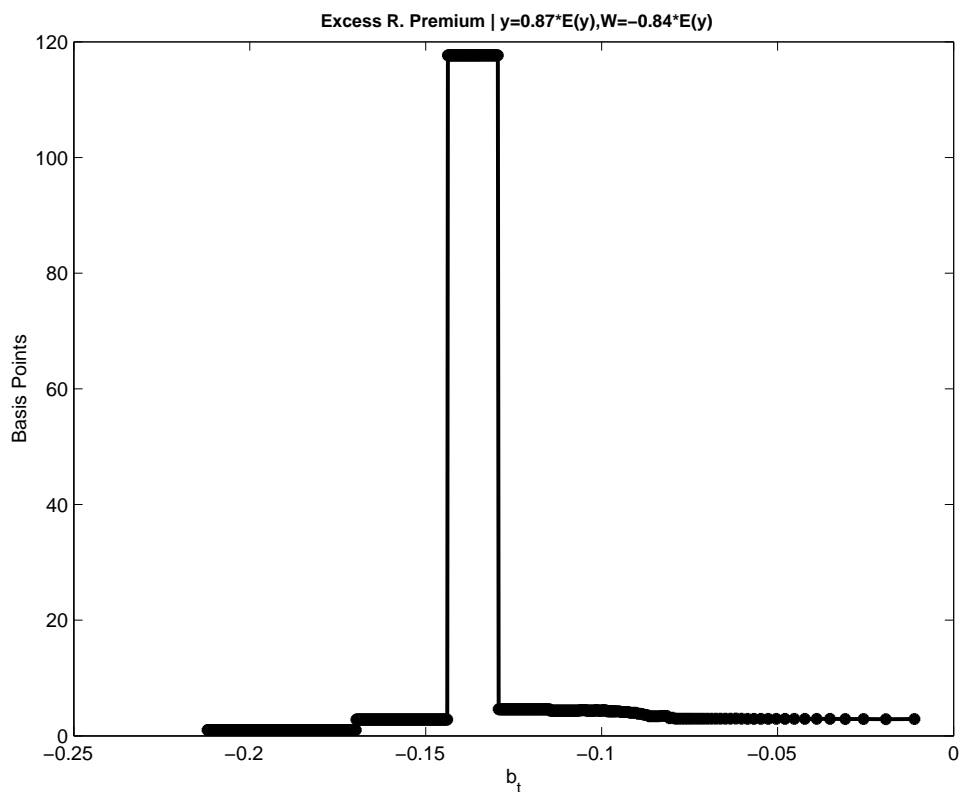


Figure 1: The Risk Premium in Sovereign Bond Prices.

## 4.2 Policy Functions

Numerical results of this exercise confirm the analytic results previously discussed. The following graphs correspond to the case in which (i) there are only transitory shocks to the income process, (ii) the probability of coming back to the credit market after defaulting is 10%, (iii) the output loss is 2% in any period in which the economy is in a default state, and (iv) the investors risk aversion is 2.

**Investors' Risk Aversion and Policy Functions** One of the implications from considering risk averse investors is that sovereign bond prices carry two type of premiums: a default probability premium and a excess risk premium. Given the level of investors' wealth,  $W$ , Figure 1 shows the *excess* premium in sovereign bonds once the premium that compensates investors for the default probability was taken in account.

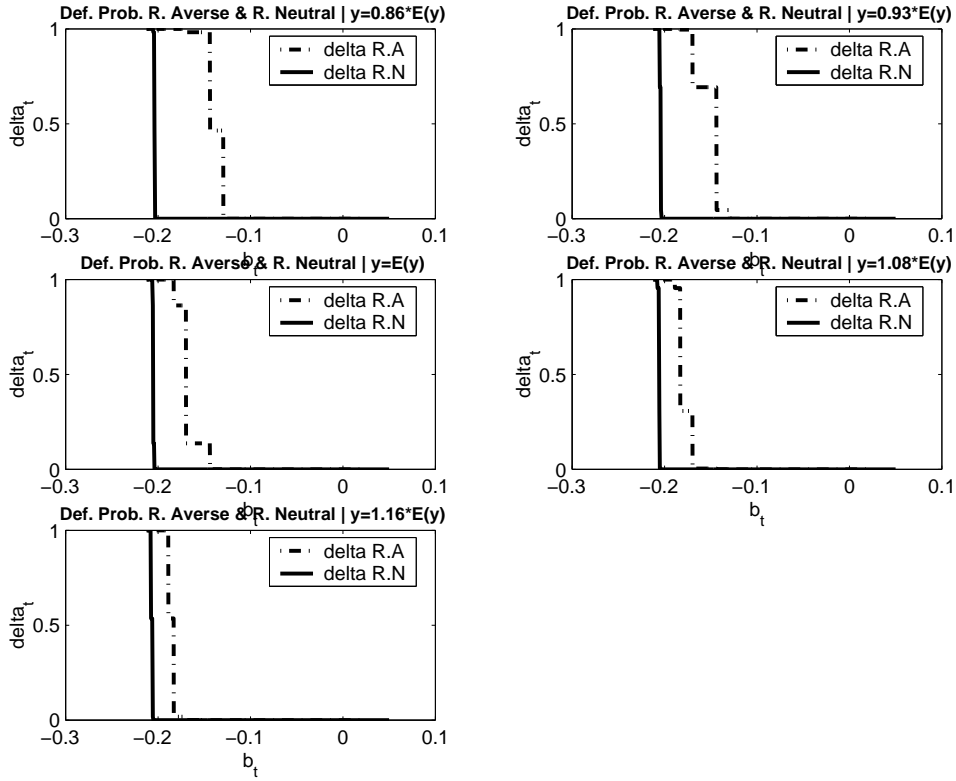


Figure 2: Default Probabilities: Risk Neutral vs. Risk Averse Investors.

The model also predicts that default risk, proxied by default probabilities, is always higher for more risk averse investors. Figure 2 shows default probability functions for two identical economies that trade with two different type of investors, one risk neutral and the other risk adverse. For a given level of wealth, Figure 2 shows that when investors are risk averse, the probability of default is greater than or equal to the probability of default associated with the same levels of debt when investors are risk neutral. This result holds for all realizations of the economy’s endowment and all levels of debt.

**Investors’ Wealth and Policy Functions** The numerical results of the model confirm the previous analytic results: debt limits tighten when wealth falls. Table 3 shows the debt limits of the model as a function of investors’ initial wealth.

Table 3: Credit Limits and Investor’s Wealth

Credit Limits as a function of Investors’ Wealth										
Wealth Level	-1.07	-0.85	-0.59	-0.29	0.05	0.45	0.94	1.58	2.47	4
Credit Limit	-0.190	-0.192	-0.193	-0.194	-0.196	-0.197	-0.199	-0.201	-0.205	-0.205

### 4.3 Simulations

The business cycle statistics of the data are compared with two pairs of different specifications of the model. These two pairs are chosen because they fit the data for the given parameters.

The first pair of specifications corresponds to the case in which (i) the endowment process of the emerging economy only exhibits transitory shocks, (ii) the probability of re-entering credit markets after a default is 0.1 and (iii) there is a proportional loss of output of 2% during the periods of exclusion. (iv) The constant-relative risk aversion coefficient of the investors is set to 0 and 2 respectively. These two alternative specifications are named as “Model 1A: risk neutral investors and transitory shocks” and “Model 1B: risk averse investors and transitory shocks.”

The second pair of specifications of the model correspond to the case in which (i) the endowment process of the emerging economy exhibits shocks to the trend, (ii) the probability of re-entering credit markets after a default is 0.5 and (iii) there is a proportional loss of output of 2% during the periods of exclusion. (iv) Again the constant-relative risk aversion coefficient of the investors is set to 0 and 2 respectively. These two alternative specifications are named as “Model 2A: risk neutral investors and shocks to the trend” and “Model 2B: risk averse investors and shocks to the trend.”

All the other parameters for the simulations are as described in Table 2. The results of the simulations are shown in Table 4. The statistics in that table are the average for 1000 simulations of 60,000 periods each (i.e. 2,400 years).

The simulations presented here show that considering risk averse investors instead of risk neutral investors provides a better match to the risk premium of sovereign bond prices and its volatility as well as to the level of borrowing by emerging economies. An empirical weakness of risk neutral models is that if a proportional output loss is assumed as is the case of these simulations, then in order to match the observed time series behavior of default events, those models need to use values for the emerging economy’s discount rate which are too low—to match the time series behavior of default events risk neutral models typically require values between 0.79 and 0.89. The model in the current paper is able to use a more

Table 4: **Business Cycle Statistic: The Model and the Data.**

Statistics	Data	Model 1A	Model 1B	Model 2A	Model 2B
mean $(r - r^f)$	12.67%	0.00%	1.61%	0.00%	2.88%
std $(r - r^f)$	5.42%	0.00%	11.41%	0.00%	9.17%
mean $(-b/y)$	53.30%	20.34%	16.78%	4.28%	4.01%
Default Probability	0.74%	0.00%	1.25%	0.00%	0.94%
std $(c)/\text{std}(y)$	1.080	0.981	0.982	0.996	1.008
std $(tb/y)$	2.17%	0.49%	1.24%	0.07%	0.35%
corr $(y, r - r^f)$	-0.543	0.00	-0.256	0.00	-0.172
corr $(tb/y, y)$	-0.424	0.263	0.230	0.1904	0.1377
std $(c^{US})/\text{std}(y^{US})$	0.8163	N.A	0.059	N.A	0.022
corr $(W, r - r^f)$	-0.326	N.A	-0.438	N.A	-0.124
corr $(c, c^{US})$	0.4165	N.A	0.460	N.A	0.267
corr $(tb/y, y^{US})$	-0.1412	N.A	-0.031	N.A	0.022

standard value of 0.95. This larger discount rate allows the current model to support higher levels of debt at equilibrium—which are closer to the observed levels. Furthermore, because the risk premium in the asset prices has to be large enough to compensate the investor not only for the probability of default, but also for taking the risk of default, other things equal the model simulated here is able to account for a larger proportion of credit spreads than models with a representative risk neutral investor.

In the data the mean interest rate spread is 12.67%. In the model the mean interest rate spread is 1.61% for the model 1B and 2.88% for the model 2B. Additionally, the volatility of the spread in the data is 5.42%, in the model this volatility is 11.41% and 9.17% respectively.

While the level of the spread is not as high as the observed in the data, it is higher than the value of 0 obtained for identical economies that trade financially with risk neutral investors. It is important to note that this spread corresponds to the average for those periods in which the economy is in a repayment state, that is, when the economy is not excluded from financial markets. If the average over all periods is considered, the model generates spreads of 16.26% and 4.87% respectively.

On the other hand, considering risk averse investors tends to over-estimated the volatility of the interest rate spread. The reason for this result is that with higher levels of risk aversion the equilibrium probability of default is much higher than in the data. However, the model for identical economies that trade financially with risk neutral investors does not generate any volatility in the spread. In the data, the mean debt-to-output ratio is around 53.3%, and it is around 16.78% for the model 1B and 4.01% for the model 2B.

Overall, the results here compare favorably to previous literature: the mean spread in Arellano (2008) is 3.58% and its volatility is the 6.36%. While these values are closer to

the data than the statistics reported for the same variables in this paper, Arellano can only support a mean debt-to-output ratio of 5.95%; the parametrization of model 1B generates a value of 16.78% for this ratio which is much closer to the observed value in the data. Aguiar and Gopinath (2006) perform even better matching the debt-to-output ration, reporting a value of 19%, but their model only generates a mean interest rate spread of 1.51% with a volatility of 0.32%, both values are smaller than the ones for the model 2B.

The model introduced here also reproduces the counter-cyclical behavior of domestic interest rates. The numerical solution of the model shows that the correlation between domestic interest rates and output is around  $-0.256$  for model 1B and around  $-0.172$  for model 2B. These values of the correlation are lower than the observed value for the data  $-0.543$  but much higher than the value of 0 obtained for identical economies that trade financially with risk neutral investors.

The correlation between domestic interest rates and output found here also compares favorably with the existing literature which imposes risk neutral investors: Arellano (2008) finds a correlation between output and domestic interest rates of  $-0.29$ . Aguiar and Gopinath (2006) find a correlation between output and domestic interest rates of  $-0.03$ .

Regarding the role of investors characteristics, the simulations here suggest that for the models 1B and 2B the probability of default does not seem to be affected by the investors' wealth. The mean default probability is around 1.25% for model 1B and 0.94% for model 2B. This rates are equivalent to annual default rates of 5.1% and 3.82% or around 8.91 and 6.68 defaults each 175 years respectively. These default rates are higher than the default rate found elsewhere. For example, Reinhart et al.(2003) found an average of 5.2 defaults per defaulting country each 175 years. However,even though the results of the current model overestimate the default rate, it is important to highlight that one of the main limitations of the models in this literature is the difficulty of generating default at equilibrium when considering relatively high discount rates.

The model is also consistent with a few statistics that the previous literature cannot account for: First, taking the consumption of a developed country like the U.S. as a proxy for the investors, the data shows a positive correlation with the consumption of Argentina, and the value of this correlation is 0.4165. The model presented here delivers a correlation of 0.46 for model 1B and 0.267 for model 2B.

Second, in the data, the correlation between a measure of the investors performance, the SP500, and Argentina's interest spread is  $-0.326$ . The model generates a value for this correlation of  $-0.438$  for model 1B and 0.124 for model 2B.

Third, in the data, Argentina's trade balance and the U.S. output are negatively correlated at  $-0.141$ ; in the model the correlation between investors wealth and trade balance is  $-0.0301$  for model 1B and  $0.022$  for the model 2B.

Finally, the main quantitative weakness of the model is that it does not reproduce the negative correlation between the trade balance and the output that is observed in the data. This result does not depend on considering either transitory income shocks or shocks to the growth trend like the ones in Aguiar and Gopinath (2006).

The lack of a match of the correlation of the trade balance and the output depends on the large steepness of the price function of the sovereign bonds of the emerging economy. In the current paper, given the level of wealth of international investors, the decision of default is not very sensitive to the realization of any type of income shocks but instead highly sensitive to the quantity borrowed. These interactions result in a very steep equilibrium price function. The high steepness of the price function discussed in Aguiar and Gopinath (2006) reappears in the context of the current model due to the fact that when investors are risk averse, small changes in the borrowing level imply relatively large changes in the price of the bonds due to the "excess" risk premium that investors' risk aversion commands in comparison with the context of risk neutral investors.

A way of regaining the negative correlation between the trade balance and output would be to break the tight link between the level of borrowing and the price of the bonds. One might think that this tight link could be broken by considering an asymmetrical output loss following a default like the one in Arellano (2008). However the parameters considered in the current paper do not suggest that this is a solution.

#### 4.4 Sensitivity Analysis

From the previous results it looks like the model that better fits the data is the one with transitory shocks. Therefore the following exercise compares the business cycles statistics for five economies with transitory shocks and a probability of re-entering credit markets of 0.1.

The first economy is the economy of the model 1A in the previous simulations. The second economy is an economy trading with risk neutral investors that are credit constrained in a way in which their endogenous credit limits match the credit limits of the economy in the model 1B in the previous simulations. The third economy is an economy trading with risk averse investors who have a risk aversion parameter of 0.5. The fourth economy trades with investors having a risk aversion parameter of 1. The fifth economy trades with

Table 5: Sensitivity Analysis (Quarterly)

	1A	RNW	$\gamma_L = 0.5$	$\gamma_L = 1$	1B
Mean ( $r$ )	1.01%	1.01%	1.01%	1.01%	2.61%
Std. Dev. ( $r$ )	0.00%	0.24%	1.63%	1.72%	11.42%
Corr. ( $r, y$ )	-0.00	-0.01	-0.03	-0.04	-0.26
Corr. ( $r, W$ )		0.04	-0.24	-0.16	-0.44
Std. Dev. ( $c$ ) / Std. Dev. ( $y$ )	0.981	0.973	0.983	1.005	0.983
Corr. ( $c, y$ )	0.997	0.994	0.996	0.992	0.976
Corr. ( $c, W$ )		-0.001	0.009	0.008	0.146
Std. ( $c^L$ ) / Std. ( $c^L$ )		0.04	0.12	0.07	0.06
Corr. ( $c^L, y$ )		0.35	0.43	0.10	0.46
Corr. ( $c^L, W$ )		0.15	0.08	0.11	0.13
Std. Dev. ( $tb/y$ )	0.49%	0.62%	0.53%	0.73%	1.24%
Corr. ( $tb/y, y$ )	0.26	0.29	0.23	0.03	0.23
Corr. ( $tb/y, W$ )		0.01	0.05	0.00	-0.03
Mean. Default Probability	0.00%	0.00%	0.03%	0.03%	1.25%
Mean Debt to Output	20.34%	12.83%	17.91%	13.08%	16.78%

investors having a risk aversion parameter of 2, so this is the economy of the model 1B in the previous simulations.

The comparison of these five economies in Table 5 suggests that the probability of observing default in the model increases when the assumption of risk neutrality of investors is relaxed. Specifically, for a discount rate of 0.95, while the model with risk neutral investors barely generates a positive probability of default, the model with risk averse investors might produce an (excessively) high default probability. Increasing risk aversion of investors also makes consumption and the domestic interest rate of the economy relatively more sensitive to changes in the wealth of international investors.

In comparing the results in this paper with the results for an economy that trades with risk neutral investors that are financially constrained, we see that the importance of investor's wealth in the determination of sovereign spreads and emerging economies business cycles is due to the fact that the investors preferences exhibit DARA and not due to the fact that these agents might be financially constrained. The fitting of the data by the model with risk averse investors is much superior to the fitting of the data with financially constrained risk neutral investors.

## 5 Conclusion

This paper presents a stochastic dynamic general equilibrium model of default risk that endogenizes the role of external factors in the determination of small open economies' incentives to default, sovereign bond prices, capital flows and default episodes.

The empirical literature on international finance presents evidence that points to a very relevant role for investors' characteristics—risk aversion and wealth—in the determination of sovereign credit spreads and capital flows to emerging economies. The model in this paper is the first model with endogenous default risk that can account for these empirical findings. By relaxing the assumption of risk neutrality on the side of international investors and assuming that the preferences of these agents exhibit decreasing absolute risk aversion, this model generates a link between international investors' characteristics and emerging economies' sovereign credit markets.

Therefore, the contribution of the paper is twofold. First, the paper qualitatively and quantitatively characterizes the role of investors' characteristics in the determination of small open economies' optimal plans when international credit contracts cannot be enforced. Second, the paper presents a theoretical framework that is extended in a companion paper (Lizarazo (2009)) to a multi-country setup to study endogenous financial links across countries with common investors. This extension can explain endogenously the occurrence of contagion in sovereign debt markets of emerging economies.

Regarding the role of investors' characteristics, the analytical results of this model establish that default risk increases with investors' risk aversion and decreases with investors' wealth. Investors' characteristics have the opposite effect on capital flows. Capital flows decrease with investors' risk aversion and increase with investors' wealth. As a consequence, credit limits are tighter when investors are more risk averse or less wealthy.

Quantitatively, the model developed here outperforms previous models of endogenous default risk in several ways. Compared to risk-neutral models using the same parameterizations, the current model performs better at explaining sovereign yield spreads levels and equilibrium debt levels, even when investors might be financially constrained.<sup>26</sup> In comparison to those models with risk neutral investors, the present model supports a combination

---

<sup>26</sup>Other things equal, in order to generate default at equilibrium under the assumption of risk-neutral investors, models of endogenous default require a combination of two factors: (i) relatively low time invariant discount rates on the part of the emerging economy and (ii) a high number of possible realizations of the income/productivity shock of the emerging economy. This paper uses a discount factor of 0.95 and 5 possible realizations for the income shocks of the emerging economy.

of higher levels of debt at equilibrium and higher and more volatile spreads. The model is also able to replicate the counter-cyclical behavior of domestic interest rates. Finally, the model is consistent with the observed positive correlation between measures of investors performance and interest rate spreads: First, this model exhibits the expected negative correlation between investors' wealth and sovereign spreads. Second, the correlation between investors' risk appetite (given by  $-\frac{v_{c^L c^L}(c^L)}{v_{c^L}(c^L)} = \frac{\gamma^L}{c^L}$ ) and sovereign spreads has the expected negative sign.

While the model improves on explaining the behavior of prices and quantities with respect to models of the same type that do not consider investor's characteristics, the model is not without shortcomings. For example, the maximum level of debt supported at equilibrium is only around 20% of the output, which is much lower than the 53.3% average reached by Argentina at the verge of default reported in Reinhart et al.(2003). The present model could also improve upon the relationship between the trade balance and output; the model generates a positive correlation which is not consistent with the data. Finally, from a computational perspective, the inclusion of an additional state variable (the level of wealth of the investors) makes solving this problem much more intensive than the simpler model.

Nonetheless the model presented here opens the door to an important economic issue—that the creditworthiness of a country can be partially explained by factors other than the country's own fundamentals. This more general framework can shed light on a multitude of policy questions: the optimal degree of diversification of international portfolios; the appropriateness of capital controls to exclude volatile short-term flows; the role of the IMF in preventing crises; the impact of term-structure on debt markets; and the transmission of crises from debt markets to equity markets. While these questions remain to be explored, a clear message emerges from the current analysis: The consideration of risk averse investors goes a long way toward explaining sovereign bond spreads and the behavior of borrowers and investors in emerging markets.

## References

- [1] Aiyagari, R., (1994). "Uninsured Idiosyncratic Risk and Aggregate Saving." *Quarterly Journal Of Economics* 109(3), 600-684, 1994.
- [2] Aguiar, M., and Gopinath, G., (2006). "Defaultable Debt, Interest Rates and the Current Account." *Journal of International Economics* 69, 64-83, 2006.
- [3] Arellano, C., (2008). "Default Risk and Income Fluctuations in Emerging Economies." *American Economic Review* 98, 670-712, 2008.
- [4] Arora, V., and Cerisola, M., (2001). "How Does US Monetary Policy Influence Sovereign Spreads in Emerging Markets?." *IMF Staff Papers* Vol 48, No 3, International Monetary Fund.
- [5] Bai, Y., and Zhang, J.,(2006). "Financial Integration and International Risk Sharing." *Working Paper* Michigan University, 2006.
- [6] Baig, T., and Goldfajn, I.,(1998). "Financial Market Contagion in the Asian Crisis." *IMF Working Paper* WP/98/155, November 1998.
- [7] Baig, T., and Goldfajn, I.,(2000). "The Russian default and the Contagion to Brazil." *IMF Working Paper* WP/00/160, October 2000.
- [8] Bekaert, G., Harvey, C., and Lumsdaine, R., (2002). "Thedynamics of emerging market equity flows." *Journal of International Money and Finance* 21(3), 295-350, 2002.
- [9] Broner, F., Lorenzoni, G., and Schmulker, S., (2005). "Why do emerging economies borrow short term?." *Working Paper* MIT, 2005.
- [10] Cantor, R., and Packer, F, (1996). "Determinants and Impact of Sovereign Credit Ratings." *Economic Policy Review* October, 1996. Federal Reserve Bank of New York.
- [11] Chatterjee, S., Corbae, D., Nakajima, M., and Rios-Rull, J., (2007). "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default." *Econometrica* 75, 1525-1589, 2007.
- [12] Cline, W., and Barnes, K., (1997). "Spreads and Risk in Emerging Market Lending." *Working Paper* 97-1, December 1997. Institute of International Finance.
- [13] Cuadra, G., and Sapriza, H., (2008). "Sovereign default, interest rates and political uncertainty in emerging markets." *Journal of International Economics* , 2008 (forthcoming).

- [14] Cunningham, A., Dixon, L., and Hayes, S., (2001). "Analyzing Yield Spreads on Emerging Market Sovereign Bonds." *Financial Stability Review*, December 2001.
- [15] Eaton, J., and Gersovitz, M.,(1981). "Debt with Potential Repudiation: Theoretical and Empirical Analysis." *Review of Economic Studies* Vol XLVIII, 289-309.
- [16] Eichengreen, B., and Mody, A., (2002). "Interest Rates in the North and Capital Flows to the South: Is There a Missing Link? " *International Finance* Vol 1, Issue 1, 35-57, 2002.
- [17] Ferruci, G., Herzberg, V., Soussa, F., Taylor, A., (2004). "Understanding Capital Flows to Emerging Market Economies." *Financial Stability Review: June 2004*.
- [18] FitzGerald, V., and Krolzig, D., (2003). "Modeling the Demand for Emerging Market Assets." *Working Paper*, April (2003).
- [19] Forbes, K., and Rigobon, R., (1999). "No contagion only Interdependence: Measuring Stock Market Comovements ". *NBER Working Paper 7267*, July 1999
- [20] Gelos, G., Sahay R., and Sandleris, G., (2004). "Sovereign Borrowing in Developing Countries: What Determines Market Access?." *Working Paper*, 04/221, IMF, 2004.
- [21] Goldberg, L., (2006). "The International Exposure of US Banks: Europe and Latin American Compare ." *NBER Working Paper*, 11365, May 2005.
- [22] Hatchondo, J., Martinez, L. and Saprizo H., (2008) "Heterogeneous Borrowers in Quantitative Models of Sovereign Default " *International Economic Review (forthcoming)*, (2008).
- [23] Hernandez, L., Mellado, P., and Valdes, R., (2001). "Determinants of Private capital flows in the 1970s and 1990s: Is there evidence of Contagion?." *IMF Working Paper* WP/01/64, May 2001.
- [24] Hernandez, L., and Valdes, R., (2001). "What drives Contagion: Trade, Neighborhood, or Financial links " *IMF Working Paper* WP/01/29, March 2001.
- [25] Kamin, S. and von Kleist, K., (1999). "The Evolution and Determinants of Emerging Market Credit Spreads in the 1990s." *Working Paper (1999)*. Bank for International Settlements and Federal Reserve Board.
- [26] Kaminsky, G., and Reinhart, C., (1998). "Financial Crises in Asia and Latin America: Then and Now." *American Economic Review* Vol 88, Issue 2, Papers and Proceedings

of the Hundred and Tenth Annual Meeting of the American Economic Association, May 1998, 444-448.

- [27] Kaminsky, G., and Reinhart, C., (2000). "On crises contagion and confusion." *Journal of International Economics* 51(1), 145-168, 2000.
- [28] Kaminsky, G., Lyons, R., and Schmukler, S., (2001). "Mutual Fund Investment In emerging Markets: An Overview." *The World Bank Economic Review*, Vol 15, No 2 315-340, 2001.
- [29] Kletzer, K. and Wright, B.,(2000) "Sovereign Debt as Intertemporal Barter" *American Economic Review* 90(3), 621-639, 2000.
- [30] Lizarazo, S., (2009) "Contagion of Financial Crises in Sovereign Debt Markets." *Working Paper* ITAM, January 2009.
- [31] Mody, A., and Taylor, M., (2003). "The High Yield Spread as a Predictor of Real Economic Activity: Evidence of a Financial Accelerator for the United States." *IMF Staff papers* 50(3), 373-402, 2003.
- [32] Neumeyer, P., and Perri, F., (2005). "Business Cycles in Emerging Economies: The Role of Interest Rates." *Journal of Monetary Economics* 52/2, 345-380, March 2005.
- [33] Paasche, B., and Zin, S., (2001). "Competition and Intervention in Sovereign Debt Markets." *NBER Working Paper* 8679, December 2001.
- [34] Reinhart, C., (2002). "Default, Currency Crises and Sovereign Credit Ratings." *NBER Working Paper* 8738, January 2002.
- [35] Reinhart, C., Calvo, G., Fernandez-Arias, E., and Talvi, E., (2001). "The Growth-Interest Rate Cycle in the United States and its Consequences for Emerging Markets " *RES Working Papers* 4279, Inter-American Development Bank, Research Department, 2001.
- [36] Reinhart, C., Rogoff, K., and Savastano, M., (2003). "Debt Intolerance." *NBER Working Paper* 9908, August 2003.
- [37] Uribe, M., and Yue, V., (2006). "Country Spreads and Emerging Countries: Who Drives Whom?." *Journal of International Economics* 69, 6-36, 2006.
- [38] Valdes, R., (1996). "Emerging Markets Contagion: Evidence and theory." . Banco Central de Chile. *Documentos de Trabajo del Banco Central*.

- [39] Van Rijckeghem, C. and Weder, B., (2001). "Sources of Contagion: Finance or Trade?." *Journal of International Economics* 54(2), 293-308, 2001
- [40] Warther, V., (1995). "Aggregate Mutual Funds Flows and Security Returns." *Journal of Financial Economics*, 39, 209-235
- [41] Westphalen, M., (2001). "The Determinants of Sovereign Bond Spreads Changes." *Working Paper*, Universite de Lausanne, and Fame, November (2001).
- [42] Wright, M., (2002). "Reputations and Sovereign Debt." *Working Paper*, September 2002.
- [43] Yue, V., (2006). "Sovereign Default and Debt Renegotiation." *Working Paper*, New York University, November 2006.

# Appendix 1

## Proofs

The proofs that follow assume the extreme case of permanent exclusion of credit markets after default by the emerging economy. This assumption simplifies the proofs because in this case the value of the value function of default is independent of the investor's degree of risk aversion and wealth level. However the results can be generalized to the case of temporary exclusion using the following argument: In the more general case of temporary exclusion after a default, the value of the value function of default depends on the risk aversion and the wealth of the investors but with a discount: the future periods in which the economy might re-enter the credit market are discounted by the economy's discount factor  $\beta$  and by the probability of re-entering credit markets  $\tau$ , both of which are lower than 1. This discounting of the future implies that in response to changes in the coefficient of risk aversion or changes in the level of wealth of the investors, the value function of repayment must respond more strongly than the value function of default. Therefore the results of these proofs will also hold for the more general case using the argument of continuity with respect to the probability of re-entering credit markets that can vary between 0 and 1.

**Proposition 2** *For any state of the world  $s$ , the emerging economies' incentives to default are stronger in a world with a more risk averse representative investor than in a world with a less risk averse representative lender.*

**Proof.** The investor's value function can be written as

$$V^L = E \sum_{t=\tau}^{\infty} \beta^{t-\tau} v \left( X + \theta_t^{TB} - q^f \theta_{t+1}^{TB} + d_t [\theta_t - q_t \theta_{t+1}] \right).$$

Considering the case in which the economy has not defaulted in the current period (otherwise the investor will not invest in this economy in this period) and assuming an interior solution for the allocation to the emerging economy's asset

$$\phi(\theta') = E \{ -qv_c(c_L(\theta')) + \beta v_c(c'_L(\theta')) d' \} = 0.$$

If the periodic utility of the international investor is of the CRRA type and  $\gamma_1^L < \gamma_2^L$ , then there exists a concave function  $\psi(\cdot)$  such that  $v_2(c; \gamma_2^L) = \psi(v_1(c; \gamma_1^L))$ . If  $\theta'_1$  is the optimal allocation when  $\gamma^L = \gamma_1^L$ , and  $\theta'_2$  is the optimal allocation when  $\gamma^L = \gamma_2^L$  then it

holds that

$$\begin{aligned}\phi_1(\theta'_1) &= E \{ -qv_{1,c}(c_L(\theta'_1)) + \beta v_{1,c}(c'_L(\theta'_1)) d' \} = 0. \\ \phi_2(\theta'_2) &= E \{ -qv_{2,c}(c_L(\theta'_2)) + \beta v_{2,c}(c'_L(\theta'_2)) d' \} = 0.\end{aligned}$$

Using  $v_2(c; \gamma_2^L) = \psi(v_1(c; \gamma_2^L))$  it is possible to define

$$\phi_2(\theta'_1) = E\psi' [v_1(\theta'_1)] \{ -qv_{1,c}(c_L(\theta'_1)) + \beta v_{1,c}(c'_L(\theta'_1)) d' \} < 0.$$

The last inequality comes from the fact that  $\psi'(\cdot)$  is positive and decreasing. The inclusion of this function in the previous equation implies that  $\phi_2(\theta'_1)$  is lower than  $\phi_2(\theta'_2)$  because  $\psi'(\cdot)$  gives little weight to the realizations of  $d' = 1$ , and high weight to the realizations of  $d' = 0$ . Therefore

$$\phi_2(\theta'_2) > \phi_2(\theta'_1).$$

The concavity of  $V^L(\cdot)$  implies that given  $q$  and the risk of default (represented by the expected realizations of  $d'$ )  $\phi(\theta')$  is a decreasing function, and as consequence

$$\theta'_2 < \theta'_1$$

which in equilibrium implies  $b'_2 < b'_1$ .

Then for any state of the world  $s$  and taking as given  $q$  and the risk of default ( $\delta$ ), a higher degree of risk aversion of the investor would result in this agent allocating a lower proportion of her portfolio to the economy's sovereign bonds. Therefore, when the investor is less risk averse there are financial contracts that are available to the emerging economy that are not available when the investor is more risk averse. Consequently, given  $q$  and  $\delta$ ,

$$V_1^C(s; \gamma_1^L) \geq V_2^C(s; \gamma_2^L).$$

Because the utility of autarky for the emerging economy does not depend on the investor's risk aversion, it is clear that if for some state of the world,  $s$ , default is optimal when  $\gamma^L = \gamma_1^L$ , then for the same state of the world default would be optimal when  $\gamma^L = \gamma_2^L$ . Additionally, because incentives to default would be higher whenever  $\gamma^L = \gamma_2^L$ , then  $\gamma^L = \gamma_1^L$  implies that at equilibrium  $\delta(s, b'; \gamma_2^L) > \delta(s, b'; \gamma_1^L)$ , and therefore  $q(s, b'; \gamma_2^L) < q(s, b'; \gamma_1^L)$ . Then, unambiguously for all states of the world, the emerging economy faces stronger incentives to default when the investor is more risk averse. ■

**Proposition 3** *Default sets are shrinking in assets of the representative investor. For all  $W_1 < W_2$ , if default is optimal for  $b$  in some states  $y$ , given  $W_2$ , then default will be optimal for  $b$  for the same states  $y$ , given  $W_1$ . Therefore  $D(b | W_2) \subseteq D(b | W_1)$*

**Proof.** Proof: From Equation (6), if  $W_1 < W_2$ , then for any given  $q$  and taking as given the level of default risk,

$$b'_2 < b'_1.$$

This inequality holds because decreasing absolute risk aversion implies that  $v(X + W_1 - q^f \theta_{t+1}^{TB} - d_t q_t \theta_{t+1})$  is a concave transformation of  $v(X + W_2 - q^f \theta_{t+1}^{TB} - d_t q_t \theta_{t+1})$  (see Proposition 6.C.3 of Mas-Colell and Whinston); so if  $\theta'_1$  is the optimal allocation when  $W = W_1$ , and  $\theta'_2$  is the optimal allocation when  $W = W_2$ , and defining  $v_1(\theta_{1,t+1}) = v(X + W_1 - q^f \theta_{t+1}^{TB} - d_t q_t \theta_{1,t+1})$ , and  $v_2(\theta_{2,t+1}) = v(X + W_2 - q^f \theta_{t+1}^{TB} - d_t q_t \theta_{2,t+1})$  then

$$\begin{aligned}\phi_1(\theta'_1) &= E \{ -qv_{1,c}(c_L(\theta'_1)) + \beta v_{1,c}(c'_L(\theta'_1)) d' \} = 0, \\ \phi_2(\theta'_2) &= E \{ -qv_{2,c}(c_L(\theta'_2)) + \beta v_{2,c}(c'_L(\theta'_2)) d' \} = 0,\end{aligned}$$

and because  $v_1(\theta_{t+1}) = \psi(v_2(\theta_{t+1}))$

$$\phi_1(\theta'_2) = E\psi' [v_2(\theta'_2)] \{ -qv_{2,c}(c_L(\theta'_2)) + \beta v_{2,c}(c'_L(\theta'_2)) d' \} < 0.$$

The inequality comes from the fact that  $\psi'(\cdot)$  is positive and decreasing. The inclusion of this function in the previous equation implies that  $\phi_1(\theta'_2)$  is lower than  $\phi_1(\theta'_1)$  because  $\psi'(\cdot)$  gives little weight to realizations of  $d' = 1$ , and high weight to realizations of  $d' = 0$ . Therefore

$$\phi_1(\theta'_2) < \phi_1(\theta'_1).$$

The concavity of  $V^L(\cdot)$  implies that given  $q$  and the risk of default (represented by the expected realizations of  $d'$ ),  $\phi(\theta')$  is a decreasing function, and as consequence

$$\theta'_2 > \theta'_1$$

which in equilibrium implies  $b'_2 < b'_1$ .

For the emerging economy, when  $W$  increases, the best available contract under this state of the world is given by

$$\{q(s_2; b'(s_2)), b'(s_2)\}$$

(The representative agent of the emerging economy chooses  $b'(s_2)$  knowing that the equilibrium price of the sovereign bonds is  $q(s_2; b'(s_2))$ ). Then for any given level of bond prices, the emerging economy is able to borrow at least as much when the wealth of the investors is  $W_2$ , as it would be able to borrow when the wealth of those investors is  $W_1$ . Because the representative agent of the emerging economy chooses  $\{q(s_2; b'(s_2)), b'(s_2)\}$  even when

$\{q(s_1; b'(s_1)), b'(s_1)\}$  is available in state  $s_2$ , it is clear that because the representative agent is maximizing utility

$$V^c(s_2) > V^c(s_1).$$

Given that when the state of the world is  $s_2$  default is the optimal choice, it must hold

$$V^D(s_2) > V^c(s_2) > V^c(s_1)$$

which implies that if default is optimal for  $b$  in some states  $y$ , given  $W_2$ , then default is optimal for the same states given  $W_1$ . ■

## Appendix 2

### Solution Method

The solution methodology proceeds as follows. Initially, the state space of the model is discretized for each of the state variables of the model,  $b, y$ , and  $W$ . The continuous stochastic endowment process is approximated with a discrete Markov chain that allows for 5 possible realizations of the original process distribution. This approximation is done using the methodology of Hussey and Tauchen (1991). For the emerging economy's debt position,  $b$ , the asset space takes 600 possible discrete values. Finally, investors' wealth,  $W$ , takes 10 possible discrete values. By interpolating over the grid points on  $W$ , the solution algorithm allows a de facto continuous range for  $W$ .

The solution algorithm has the following steps:

- (i) Make an initial guess for the emerging economy's value function,  $V^0(s)$ , next period asset position,  $b'^0(s)$ , default/repayment decision  $d^0(s)$ , and *equilibrium* price function  $q^{APC,(0)}(s)$ . The initial guesses are the value function, the policy function functions and the equilibrium price function that result from an analogous model with risk neutral investors  $V^{RN,0}(s)$ ,  $b'^{RN,0}(s)$ ,  $d^{RN,0}(s)$ , and  $q^{RN,(0)}(y; b'(y))$  respectively.
- (ii) Taking  $b'^{*,(-i)}(s)$ ,  $d^{*,(-i)}(s)$  and  $q^{APC(-i)}(s)$  as given, and assuming equilibrium in emerging credit markets given by

$$\theta'^{*,(i)}(s) = \begin{cases} b'^{*,(-i)}(s) & \text{if } b'^{*,(-i)}(s) < 0 \\ 0 & \text{if } b'^{*,(-i)}(s) \geq 0 \end{cases},$$

iterate on the representative investor's Bellman equation (4) to solve for the optimal value function  $V^{L(i)}(s)$  and the optimal policy functions  $W'^{*,(i)}(s)$ .

(iii) Iterate on the emerging economy's Bellman equation (11) to solve for the optimal value function  $V^{(i)}(s)$ , the optimal policy functions  $b^{*,(i)}(s)$ , and  $d^{*,(i)}(s)$  and the corresponding equilibrium price function  $q^{EE(i)}(s; b^{(i)}(s))$ . This iteration involves the following sub-steps:

(a) Take  $q^{APC,(-i)}(s)$  and  $W^{*,(i)}(s)$  as given to compute  $c_L^{(i)}(s; b')$ .

(b) Given  $c_L^{(i)}(s; b')$  and  $W^{*,(i)}(s)$ , compute

$$A^{(i)}(s, b') = \beta_L \int \left(c^{L'}\right)^{-\gamma^L} f\left(y' \mid y, W^{*,(i)}(s)\right) dy'$$

(c) For any  $s, b'$  solve for  $q^{(i)}(s, b')$  by solving the non-linear equation on  $q^{(i)}(s, b')$  that is derived from (6):

$$q(s, b')^{-\gamma^L} - b' A^{(i)}(s, b') q(s, b') - \tilde{c}_L^{(i)}(s; b') A^{(i)}(s, b') = 0$$

where  $\tilde{c}_L^{(i)}(s; b') = X + W - W^{*,(i)} q^f - b' q^f$ .

(d) For any  $s, b'$  given  $W^{*,(i)}(s)$  compute

$$\beta \int V^{C(i)}(s; b') f\left(y' \mid y, W^{*,(i)}(s)\right) dy'.$$

(e) Maximize

$$u(y + b - b' q(s, b')) + \beta \int V^{C(i)}(s; b') f\left(y' \mid y, W^{*,(i)}(s)\right) dy'$$

with respect to  $b'$  to find  $V^{C(i)}(s)$  and the associated  $b^{*,(i)}(s)$  and  $q^{(i)}(s, b^{(i)}(s))$ .

(f) Determine  $d^{*,(i)}(s)$  by comparing  $V^{C(i)}(s)$  to  $V^D$ .

(g) Determine the equilibrium price of bonds by setting

$$q^{EE(i)}(s; b^{(i)}(s)) = \begin{cases} q^{(i)}(s, b^{(i)}(s)) & \text{if } d^{(i)}(s) = 1 \\ 0 & \text{otherwise} \end{cases}.$$

(iv) If  $|q^{EE(i)}(s; b^{(i)}(s)) - q^{APC,(-i)}(s; b^{(-i)}(s))| < \varepsilon$  stop. Otherwise, set  $q^{APC,(i)}(s; b^{(i)}(s)) = q^{EE(i)}(s; b^{(i)}(s))$ , and repeat steps 2 to 4.