BAYESIAN CLAIMS RESERVING

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Abstract

A brief historical review of the use of Bayesian methods in actuarial science is first given, in order to establish a framework for the note. Next, Bayesian models are described in general terms, emphasizing their main advantages: a) they formally allow the actuary to incorporate prior non-sample information and, b) complete distributions are used in the analyses. Some examples are given of their use in claims reserving.

KEY WORDS: Bayesian inference, Claims Reserving, General Insurance, MCMC, Monte Carlo, Simulation.

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Overview

The use of Bayesian concepts and techniques in actuarial science dates back to the foundations of empirical Bayes credibility, [(2)], [(5)], although apparently it was Lundberg who first realized the importance of Bayes procedures for experience rating in 1940, [(17)]. To date, Bayesian methodology is used in various areas within actuarial science, among them loss reserving, [(19)], [(20)]. The earliest explicit use of Bayesian methods to estimate for can be found in [(13)] and [(30)], although there may be some implicit uses of Bayesian methods in claims reserving through the application of Credibility methods for this purpose, [(11)].

Claims reserving methods are usually classified as stochastic or non-stochastic (deterministic), depending on whether or not they allow for random variation. Bayesian methods fall within the first class, [(9)], [(12)], [(29)], [(32)]. Being stochastic, they allow the actuary to carry out statistical inference of reserve estimates as opposed to deterministic models, like the traditional chain-ladder. As an inference process, the Bayesian approach is an alternative to classical or frequentist statistical inference. Bayesian methods have some characteristics that make them particularly attractive for their use in actuarial practice, specifically in claims reserving.

First, they allow the actuary to formally incorporate expert or existing prior information. Very frequently in actuarial science one has considerable expert, or prior, information. The latter can be in the form of global or industry-wide information (experience) or in the form of tables. In this respect it is indeed surprising that Bayesian methods have not been used more intensively up to now. There is a wealth of ‘objective’ prior information available to the actuary. In fact, the ‘structure distribution’ frequently used was originally formulated in a Bayesian framework, [(5)].

The second, advantage of Bayesian methods is that the analysis is always done by means of the complete probability distribution for the quantities of interest, either the parameters, or the future values of a random variable. In addition to point estimates we can obtain a wealth of information. Actuarial science is a field where adequate understanding and knowledge of the complete distribution is essential: in addition to expected values we are usually looking at certain characteristics of probability distributions, e.g. ruin probability, extreme values, value at risk (VaR), and so on.

As mentioned in [(9)] “there is little in the actuarial literature which considers the predictive distribution of reserve outcomes; to date the focus has been on estimating variability using prediction errors. [It] is difficult to obtain analytically…” these distributions, that take into account both the process variability and the estimation variability. Bayesian models automatically account for all the uncertainty in the parameters. They allow the actuary to provide not only point estimates of the required reserves, and measures of dispersion such as the variance, but also the complete distribution for the reserves as well. This makes it feasible to compute other risk measures.

These distributions are particularly relevant in order to compute the probability of extreme values, especially if use of the normal approximation is not warranted. In many real/life situations the distribution is clearly skewed. Confidence intervals obtained with Normal approximations can then be very different from exact ones. The specific form of the claims distribution is automatically incorporated when using Bayesian methods, whether analytically or numerically. One advantage of full Bayesian methods is that the posterior distribution for the parameters is essentially the exact distribution, i.e. it is true given the specific data used in its derivation.

Although in many situations it is possible to obtain analytic expressions for the distributions involved, we frequently have to use numerical or simulation methods. Hence one cause, probably the main one, of the low usage of Bayesian methods up to now, has been the fact that closed analytical forms were not always available and numerical approaches were too cumbersome to carry out. However, the availability of software that allows one to obtain the posterior or predictive distributions by direct Monte Carlo methods, or by Markov chain Monte Carlo (MCMC), has opened a broad area of opportunities for the applications of these methods in actuarial science.

Some Notation

Most methods make the assumptions that: a) the time (number of periods) it takes for the claims to be completely paid is fixed and known; b) the proportion of claims payable in the *t*-th development period is the same for all periods of origin; and c)quantities relating to different occurrence years are independent, [(11)], [(29)].

Let X_{it} = number (or amount) of events (claims) in the *t*-th development year corresponding to year of origin (or accident year) *i*. Thus we have a $k{\times}k$ matrix $\{X_{it}; i = 1,...,k,\ t = 1,...,k\}$, where k = maximum number of years (sub periods) it takes to completely pay out the total number (or amount) of claims corresponding to a given exposure year. This matrix is usually split into a set of known or observed variables (the upper left hand part) and a set of variables whose values are to be predicted (the lower right hand side). Thus we know the values of X_{it} $i = 1,...,k,\ t = 1,...,k,$ for $i + t \le k + 1$, and the triangle of known values is the typical run-off triangle used in claims reserving, Table 1, [(21)].

Table 1

year of				development year			
origin	1	2	......	t	...	k-1	k
1	X_{11}	X_{12}	...	X_{1t}		$X_{1,k-1}$	X_{1k}
2	X_{21}	X_{22}	...	X_{2t}		$X_{2,k-1}$	-
3	X_{31}	X_{32}	...	X_{3t}		-	
:						-	-
k-1	$X_{k-1,1}$	$X_{k-1,2}$				-	-
k	X_{k1}	-					-

Bayesian Models

For a general discussion on Bayesian theory and methods see [(3)], [(4)], [(23)] and [(34)]. For other applications of Bayesian methods in actuarial science see [(15)], [(19)], [(20)] and [(25)]. Bayesian analysis of claims reserves can be found in [(1)], [(10)], [(13)], [(14)] and [(22)].

If the random variables X_{it}, $i = 1,...,k;$ $t = 1,...,k,$ denote claim figures (amount, loss ratios, claim frequencies, etc.) the (observed) run-off triangle has the structure given in Table 1. The unobserved variables (the lower triangle) must be predicted in order to estimate the reserves. Let $f(x_{it} \mid \underline{\theta})$ be the corresponding density function, where $\underline{\theta}$ is a vector of parameters. Then $L(\underline{\theta} \mid \underline{x}) = \prod_{i+t \le k+1} f(x_{it} \mid \underline{\theta})$ is the likelihood function for the parameters given the data in the upper portion of the triangle. Available information on the parameters, $\underline{\theta}$, is incorporated through a prior distribution $\pi(\underline{\theta})$ that must be modeled by the actuary. It is equivalent to the structure distribution. This is then combined with the likelihood function via Bayes' Theorem to obtain a posterior distribution for the parameters, $f(\underline{\theta} \mid \underline{x})$, as follows: $f(\underline{\theta} \mid \underline{x}) \propto L(\underline{\theta} \mid \underline{x})\pi(\underline{\theta})$, where $\propto$ indicates proportionality. When interest centers on inference about the parameters it is carried out using $f(\underline{\theta} \mid \underline{x})$. When interest is on prediction, as in loss reserving, then the past (known) data in the upper portion of the triangle, X_{it} for $i + t \le k + 1$, are used to predict the observations in the lower triangle z_{it} by means of the posterior predictive distribution that is defined as

$$f(z_{it} \mid \underset{\sim}{x}) = \int f(z_{it} \mid \underset{\sim}{\theta}) f(\underset{\sim}{\theta} \mid \underset{\sim}{x}) d\underset{\sim}{\theta}, \quad i = 1,...,k, \quad t = 1,...,k, \quad with \quad i + t > k + 1.$$

In claims reserving, the benefit of the Bayesian approach is in providing the decision maker with a posterior predictive distribution for every entry in the lower portion of the run-off triangle and, consequently, for any function of them. One such function could be the sum of their expected values for one given year of origin *i*, i.e. an estimate of the required claims reserves corresponding to that year:

$$R_i = \sum_{t > k-i+1} E(Z_{it} \mid D), \quad i = 2,\ldots,k.$$

Adequate understanding and knowledge of the complete distribution is essential. It allows the actuary to assess the required reserves in terms not only of expected values.

A standard measure of variability is prediction error. In claims reserving it may be defined as the standard deviation of the distribution of reserves. In the Bayesian context the usual measure of variability is the standard deviation of the predictive distribution of the reserves. This is a natural way of doing analysis in the Bayesian approach, [(1)], [(9)].

Hence, besides the usual modeling process, the actuary has two important additional tasks to carry out when using Bayesian methods:

a) Specifying the prior distribution for the parameters in the model
b) Computing the resulting posterior or predictive distribution and any of its characteristics.

Prior Distribution

This first one of these tasks is not foreign to actuarial practice. For example, in the traditional Bornhuetter-Ferguson method explicit use is made of perfect prior (expert) knowledge of ‘row’ parameters. An external initial estimate of ultimate claims is used with the development factors of the chain-ladder technique (or others) to estimate outstanding claims. This is clearly well suited for the application of Bayesian methods when the prior knowledge about the ‘row’ parameters is not perfect and may be modeled by a probability distribution. This use of external information to provide the initial estimate leads naturally to a Bayesian model, [(9)].

Bayesian models have the advantage that actuarial judgment can be incorporated through the choice of informative prior distributions. However, this may be considered as a disadvantage, since there is the risk that the approach may be mis-used. Admittedly, this approach is open to the criticism that our answers can depend on our prior, $\pi(\underline{\theta})$, and our model distributional assumptions. This should not be a conceptual stumbling block, since in the actuarial field data and experience from related problems often used to support our assumptions. The structure distribution frequently used in Credibility theory is another example of situations where actuaries use previous experience to specify a probabilistic model for the risk structure of a portfolio.

The Bayesian approach constitutes a powerful formal alternative to both deterministic and classical statistical methods when prior information is available. But they can also be used when there is no agreement on the prior information, or even when there is a total lack of it. In this last situation we can use what are known as non-informative or reference priors; the prior distribution $\pi(\underline{\theta})$ will be chosen to reflect our state of ignorance. Inference under these circumstances is known as objective Bayesian inference, [(3)]. It can also be used to avoid the criticism mentioned in the last paragraph. In many cases Bayesian methods can provide analytic closed forms for the predictive distribution of the variables involved, e.g. outstanding claims. Predictive inference is then carried out directly from this distribution. Any of its characteristics and properties, such as quantiles, can be used for this purpose. However, if the predictive distribution is not of a known type, or if it does not have a closed form, or if it has a complicated closed form, then it is possible to derive approximations using Monte Carlo (MC) simulation methods, [(6)], [(28)]. One alternative is the application of direct Monte Carlo, where the random values are generated directly from their known distribution, which is assumed to be available in an explicit form. Another alternative, when the distribution does not have a closed form, or it is a complex one, is to use Markov Chain Monte Carlo (MCMC) methods, [(25)], [(28)].

Examples

Example 1. In the run-off triangle of Table 1, let X_{it} = number of claims in *t-th* development year corresponding to year of origin (or accident year) *i*, so the available information is: $X_{it}; i=1,\ldots,k,\ t=1,\ldots,k,\ i+t \le k+1$. Let $\sum_{t=1}^{k} X_{it} = N_i$ = total number of claims for year of origin *i*, then, the Likelihood function for the unknown parameters $(n_2, n_3, \ldots, n_k, \underline{p})$, given the data, will be of the form

$$L(n_1, n_2, \ldots, n_k, \underline{p} | \underline{x}_1, \ldots, \underline{x}_k) = \prod_{i=1}^{k} f_{k-i+1}(\underline{x}_i \mid n_i, \underline{p}),$$

$i=1,\ldots, k$, where $f_k(\bullet \mid n, \underline{p})$ denotes a k-dimensional multinomial *pmf* and $\underline{p} = (p_1, \ldots, p_k)'$ is the vector of the proportions of payments in each development year, [(1)]. The vectors $\underline{x}_1 = (x_{11}, x_{12}, \ldots, x_{1k})'$, $\underline{x}_2 = (x_{21}, x_{22}, \ldots, x_{2k-1})'$, …, $\underline{x}_k = (x_{k1})$ contain the known data for the number of claims in each row of the triangle. The next step will be to specify a prior distribution for the parameters: $f(n_2, \ldots, n_k, \underline{p})$. The joint posterior distribution is then obtained as

$$f(n_2, \ldots, n_k, \underline{p} | D) \propto L(n_1, n_2, \ldots, n_k, \underline{p} | \underline{x}_1, \ldots, \underline{x}_k) \times f(n_2, \ldots, n_k, \underline{p}),$$

and it may be written in terms of the posterior distributions as:

$$f(n_2,...,n_k,\underline{p}|D)=\prod_{i=2}^{k} f(n_i|\underline{p},D)\times f(\underline{p}|D),$$

where $D=\{\underline{x}_1,\underline{x}_2,...,\underline{x}_k,n_1\}$ represents all the known information, [(1)], [(22)]. We can then use this distribution to compute the mean and/or other characteristics for any of the parameters. Notice that if in this model the quantities of interest are the total numbers of claims for each year, $(n_2,...,n_k)$, so that we can use their marginal posterior distribution, $f(n_2,...,n_k|D)$, to analyze the probabilistic behavior of the total number of claims by year of origin. However, if we want to estimate the future number of claims per cell in the lower portion of the triangle we then use the predictive distribution:

$$f(x_{it}\mid D)=\sum\int f(x_{it}\mid n_2,...,n_k,\underline{p})\,f(n_2,...,n_k,\underline{p}\mid D)d\,\underline{p},$$

for $i=1,\ldots,k;\ t=1,\ldots,k$, with $i+t>k+1$, and the summation is over $(n_2,...,n_k)$.

Example 2. Let the random variable $X_{it}>0$ represent the value of aggregate claims in the *t-th* development year of accident year i, $i,t=1,...,k$. As in the previous model, the X_{it} are known for $i+t\le k+1$. Define $Y_{it}=\log(X_{it})$. We assume in addition that

$$Y_{it}=\mu+\alpha_i+\beta_t+\varepsilon_{ij} \qquad \varepsilon_{ij}\sim N(0,\sigma^2)$$

$i=1,...,k$, $t=1,...,k$ and $i+t\le k+1$ i.e. an unbalanced two-way analysis of variance (ANOVA) model. This was originally used in claims reserving by Kremer [(16)]; see also [(1)], [(7)] and [(30)].

Thus X_{it} follows a log-normal distribution, and

$$f(y_{it} \mid \mu, \alpha_i, \beta_t, \sigma^2) \propto \frac{1}{\sigma}\exp[-\frac{1}{2\sigma^2}(y_{it} - \mu - \alpha_i - \beta_t)^2].$$

Let T_U = *(k+1)k/2* = number of cells with known claims information in the upper triangle; and T_L = *(k-1)k/2* = number of cells in the lower triangle, whose claims are unknown. If $\underline{y} = \{ y_{it} ; i,t = 1,...,k \ , i + t \leq k + 1\}$ is a T_U-dimension vector that contains all the observed values of Y_{it} , and $\underline{\theta} = (\mu, \alpha_1,..., \alpha_k, \beta_1,..., \beta_k)'$ is the $(2k+1)$ vector of parameters, then the Likelihood function can be written as

$$L(\underline{\theta}, \sigma \mid \underline{x}_1, \underline{x}_2,.., \underline{x}_k, \underline{y}) \propto \sigma^{-T_U} \exp[-\frac{1}{2\sigma^2}\sum_i^k \sum_t^k (y_{it} - \mu - \alpha_i - \beta_t)^2],$$

where the double sum in the exponent is for $i + t \leq k + 1$, i.e. the upper portion of the triangle. The actuary must next specify a prior distribution for the parameters, $f(\underline{\theta}, \sigma)$, and the joint posterior distribution is then

$$f(\underline{\theta}, \sigma \vert D) \propto L(\underline{\theta}, \sigma \mid \underline{x}_1, \underline{x}_2,.., \underline{x}_k, \underline{y}) \times f(\underline{\theta}, \sigma),$$

where $D = \{\underline{x}_1, \underline{x}_2,..., \underline{x}_k, \underline{y}\}$ represents all the known information included in the posterior distribution. The specific form of the joint posterior distribution, as well as the marginal distribution of each parameter, will depend on the choice of the prior, [(1)], [(22)], [(30)]. In this model the quantities of interest are the random variables X_{it}; $i = 1,...,k,\ t = 1,...,k,\ i + t > k + 1,$ so that it is necessary to obtain their predictive distribution:

$$f(x_{it} \mid D) = \int f(x_{it} \mid \underline{\theta}, \sigma) f(\underline{\theta}, \sigma \mid \underline{D}) d\underline{\theta} d\sigma .$$

Although under suitable conditions it also possible to derive analytic expressions for the predictive distributions, further analysis of this distribution will usually be done by some type of simulation, [(1)], [(22)], [(25)], [(30)].

Bayesian computation

In each one of the examples described above, to compute the reserves for the outstanding aggregate claims we need to estimate the values of the cells in the lower portion of the development triangle. We do this by obtaining the mean and variance of the predictive distribution. This is the second task, in addition to modeling, that we must carry out when using Bayesian methods.

For each cell we need $E(X_{it} \mid D)$, the Bayesian 'estimator'. Then the corresponding 'estimator' of outstanding claims for year of business *i* is $R_i = \sum_{t>k-i+1} E(X_{it} \mid D)$, and the Bayes 'estimator' of the variance (the predictive variance) for that same year is

$$Var(\sum_{t>k-i+1} X_{it} \mid D) = \sum_{t>k-i+1} \left[Var(X_{it} \mid D) + 2\sum_{s>t} Cov(X_{is}, X_{it} \mid D) \right]. \qquad (1)$$

In order to compute equation (1) we would need to find $Cov(X_{is}, X_{jt} \mid D)$, for each *i, j, s, t,* $i \neq j$, $t > k-i+1$ and $s>t$. Thus the covariance for each pair of elements in the lower triangle would need to be evaluated to find the variance of the reserves. These formulas can be very cumbersome to compute, [(7)], [(8)] and [(31)], and we would still not have the complete distribution. However, it may be relatively easy to obtain the distribution of the reserves by **direct** simulation, as follows: for $j = 1,..., N$, and N very large, obtain a sample of randomly generated values for claims (number or amount) in each cell of the (unobserved) lower right triangle, $x_{it}^{(j)} =$ $i = 2,.., k$ and $t > k - i + 1$, from the respective predictive distributions. These $x_{it}^{(j)}$ values will include both parameter variability and process variability. Thus for each *j* we can compute a simulated random value of the total outstanding claims

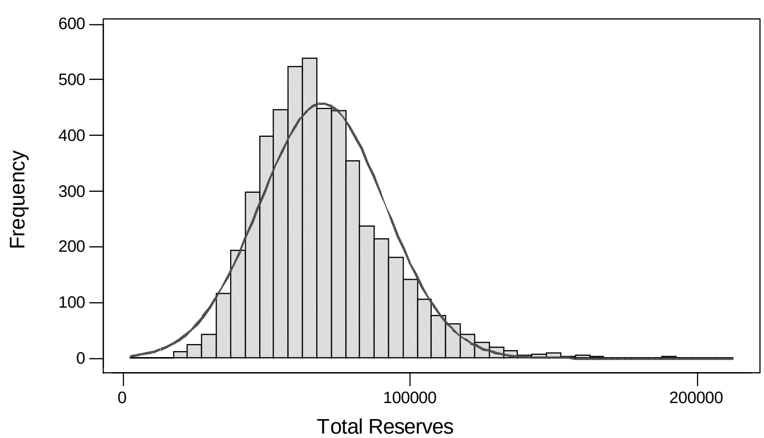
$$R^{(j)} = \sum_{i=2}^{k} \sum_{t>k-i+1} x_{it}^{(j)}, \quad j = 1,..., N.$$

These $R^{(j)}$, $j = 1,\ldots,N$, can be used to analyze the behavior of claims reserves requirements. The mean and variance can be computed as

$$\sigma_R^2 = \frac{1}{N}\sum_{j=1}^{N}\frac{(R^{(j)}-\overline{R})^2}{N} \quad and \quad \overline{R} = \frac{1}{N}\sum_{j=1}^{N} R^{(j)} \quad . \quad (2)$$

The standard deviation σ_R thus obtained is an 'estimate' for the prediction error of total claims to be paid. The simulation process has the added advantage that it is not necessary to obtain explicitly the covariances that may exist between parameters, since they are dealt with implicitly, [(1)]. Figure 1 shows an example of the distribution of reserves generated by directly simulating N=5000 values from the predictive distribution of total reserves using a model similar to the one given in Example 2, [(1)], with data from [(18)]. One can appreciate the skewness of the distribution by comparison with the overlaid Normal density.

Figure 1



The method of direct simulation outlined above is reasonable if the joint distribution of the parameters can be identified and is of a known type. When this is not the case, and the distribution may not be recognizable as a standard one, a possible solution may be found via Markov chain Monte Carlo methods.

In fact, on occasions the use of the Bayesian paradigm will not be motivated by the need to use prior information, but rather from its computational flexibility. It allows the actuary to handle complex models. As with direct simulation methods, Markov chain Monte Carlo sampling strategies can be used to generate samples from each posterior distribution of interest. A comprehensive description of their use in actuarial science can be found in [(25)]. A set of four models analogous to the examples given above is presented in [(22)] and analyzed using Markov chain Monte Carlo methods. In the discussion to that paper, it is described how those models may be implemented and analyzed using the package BUGS (Bayesian inference Using Gibbs Sampling). BUGS is a specialized software package for implementing MCMC-based analyses of full Bayesian probability models, [(26)], [(27)]. The BUGS Project Web site is found at www.mrc-bsu.cam.ac.uk/bugs. As with direct Monte Carlo simulation, and since MCMC methods provide a predictive distribution of unobserved values using simulation, it is straightforward to calculate the prediction error using equation (2).

Concluding, simulation methods do not provide parameter estimates per se, but simulated samples from the joint distribution of the parameters or future values. In the claims reserving context, a distribution of future payments in the run-off triangle is produced from the predictive distribution. The appropriate sums of the simulated predicted values can then be computed to provide predictive distributions of reserves by origin year, as well as for total reserves. The means of those distributions may be used as the best estimates. Other summary statistics can also be investigated, since the full predictive distribution is available.

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