ENDOGENOUS CAPITAL UTILIZATION AND THE AVERCH-JOHNSON EFFECT

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ABSTRACT

A variable utilization rate of capital is introduced into the intertemporal profit maximization problem of a monopoly subject to a rate of return regulatory constraint. The regulated monopoly displays the typical Averch-Johnson overcapitalization, but the impact of regulation on total services of capital is mitigated due to a lower utilization rate of the capital stock. The problem of anticipated deregulation is considered, and it is found that the firm must either “overutilize” or “underutilize” capital throughout the adjustment period.

1. INTRODUCTION

A prevalent form of regulation applied to public utilities is that of restricting the rate of return on capital. This type of constraint restricts the firm (usually a natural monopoly) to earn no more than a “fair rate of return” on its capital investment. In order to achieve this, the firm’s current profits are not allowed to exceed the regulator set return on the capital stock. A well-known result in the economics of regulation is that a monopoly subject to a rate of return constraint will have an inefficiently high capital-labor ratio. This phenomenon was analytically demonstrated for the first time by Averch and Johnson in their seminal 1962 paper. The input bias that results from the constraint has become known as the Averch-Johnson effect, henceforth referred to as the A-J effect.

A large share of the regulation literature has been devoted to elaborate on the A-J effect. Takayama (1969), presents a rigorous proof of the result and El-Hodiri and Takayama (1981) draw similar conclusions in a dynamic framework. They show that the steady state level of the capital stock increases when the rate of return constraint is imposed; thus, overcapitalization is present in the regulated firm.

Empirical verification of the A-J effect has been attempted several times with mixed results. Courville (1974) and Spann (1974), find that regulation in the electric power industry gives rise to the predicted input bias. On the other hand, Boyes (1976) and Baron and Taggart (1977) find no evidence of the A-J effect for the same industry. In order to account for these conflicting results, several explanation have been offered as to why the input bias might not be found. Baumol and Klevorick (1970), among others, suggest that the reason for the apparent absence of the effect is that the firm is not a profit maximizer. Bailey and Coleman (1971) try to explain the absence of the A-J effect by way of regulatory lags. Joskow (1974), argues that The A-J model is too simplistic to capture the intricacies of the regulatory process, and hence its implications are not very useful. Perrakis and Zerbinis (1981) show that the effect may be mitigated when uncertainty in the demand function is introduced. Dechert (1984) attempts to justify the absence of the A-J effect by assuming a convex (for small values of the labor input) production technology.

In the usual theory of factor demand capital is always utilized at a constant rate; hence, the difference between the stock of capital and the services rendered by it (the real factor of production) is irrelevant. This constant utilization rate may be normalized to unity so that the
implicit assumption is that capital is always fully utilized and as Johnson (1994) points out: “even casual observation belies this prediction”.

In this paper we solve the typical monopolistic firm profit maximization problem with the addition of a variable utilization or servicing rate of the capital stock, this endows the firm with an extra option in order to deal with the regulatory constraint. The purpose of this exercise, is to see whether there is any variation in the input bias and overcapitalization predicted by the A-J effect. We do not attempt to get involved in the regulatory process itself but rather, make the point that a variable utilization rate of capital could be accounted for variations in the magnitude of the A-J effect. As is standard in the literature, we assume a “pure user” cost of capital, so that capital depreciates more when used more intensively. In this way, we can obtain a non-trivial marginal condition for the optimal utilization rate. This assumption has been widely used within the context of capacity utilization and intensity of use. In both cases, physical depreciation is not a constant but a function of either an index of capacity utilization or a measure of intensity of use. Johnson (1994) suggests using the work-week of capital, expressed as a fraction of the maximum feasible work-week, as a measure of utilization. The construction of such a measure and its empirical estimations may be found in Shapiro (1986).

2. THE MODEL

Consider a monopolistic firm producing a single output \(Q\) using two inputs, labor \((L)\) and total capital services \((S)\). Total services of capital are defined as \(sK\), where \(K\) stands for the capital stock and \(s\) for services per unit of capital or the utilization rate. The firm is assumed to be a price taker in the labor market, and a monopsony in the capital market. The unit prices of labor and capital are \(w\) and \(p_K\) respectively. Let the revenue function be given as

\[
R(sK,L) = P(Q)Q
\]

where the function \(R\) is strictly concave and \(P\) is the inverse demand function satisfying \(P' < 0\). Capital depreciates at the variable rate, \(\delta = \delta(s)\) where \(\delta\) is a twice differentiable, increasing and convex function, and \(\delta(0) = \delta_0 \geq 0\).

The firm is subjected to following rate of return constraint:

\[
R(sK,L) - wL - p_K\delta(s)K \leq p_K^*K,
\]

where \(p^*\) is the “fair” rate of return set by the regulator. Capital evolves according to

\[
\dot{K} = I - \delta(s)K,
\]

where \(I\) stands for gross investment, the supply of which is specified by

\[
I = I(p_K),
\]

satisfying \(I' > 0\).

If \(r\) denotes the real interest rate then the firm’s intertemporal profit maximization problem can be described by
\[
\max_0^\infty \int_0^\infty (R(sK, L) - wL - p_x I)e^{-\rho t} dt ,
\]
given (3) an initial stock \(K_0\) and subject to constraint (2).

Let \(s_0\), \(K_0\) and \(L_0\) be the long run steady state values of the variables when the above problem is solved without constraint (2), then \(\rho_0 = \frac{R_0 - wL_0 - p_x \delta x_0 K_0}{p_x K_0}\) is the rate of return obtained by the unregulated firm. We assume that the firm is resting at this unregulated equilibrium when the regulator imposes constraint (2) with \(r < \rho^* < \rho_0\); thus, the initial capital stock is equal to \(K_0\).

Letting \(\lambda\) be the multiplier associated with constraint (2), the first order necessary conditions may be summarized by

\[
\begin{align*}
(5) & \quad R^* (1 - \lambda) = w(1 - \lambda) \\
(6) & \quad R^* (1 - \lambda) = p_x \delta^* (1 - \lambda) \\
(7) & \quad \dot{p}_x = p_x [r + (1 - \lambda) \delta - \lambda \rho^*] - sR_x (1 - \lambda) \\
(8) & \quad \lambda [wL + p_x K(\delta + \rho^*) - R] = 0 , \quad \lambda \geq 0
\end{align*}
\]

and the usual transversality requirements.

If at the regulated long run steady state we have \(\lambda = 0\), then the equilibrium values of the variables as obtained from (3)-(8) setting \(\dot{K} = \dot{p}_x = 0\), are the same as those in the unconstrained equilibrium. This implies that \(\rho^* = \rho_0\), which contradicts our assumption on the choice of \(\rho^*\). Thus, the constraint must be binding at the regulated long run steady state. Whenever the constraint holds with equality, (8) may be rewritten as

\[
(8^*) \quad wL + p_x K(\delta + \rho^*) - R = 0 .
\]

Assume now that when regulation takes effect at an initial time, both labor (\(L\)) and unit services (\(s\)) are chosen efficiently so that \(w = R_L\) and \(R^* = p_x \delta^*\) hold. This is always true whenever \(\lambda \neq 1\) by way of equations (5) and (6). Labor and services are now determined as implicit functions of \(K\) and \(p_x\); hence, constraint (8*) may be expressed in terms of \(p_x\) and \(K\), where

\[
\frac{dp_x}{dK} = \frac{sR_x - p_x (\rho^* + \delta)}{(\delta + \rho^*)K} .
\]

The strict concavity of the revenue function ensures that the numerator of the above expression is negative,\(^4\) this implies that the locus of points where the constraint is binding is a negatively sloped schedule in the \(K-p_x\) plane. We denote this set of points by \(C(K, p_x) = 0\). At every point above this locus the constraint (2) holds with strict inequality so that \(\lambda = 0\) and the dynamics in that region are identical to those of the unconstrained case. Mainly, they are determined by the following system of differential equations:

\[
\begin{align*}
(9) & \quad \dot{K} = I(p_x) - \delta (s) K \quad \text{and} \\
& \quad \dot{p}_x = p_x (r + \delta(s)) - sR_x .
\end{align*}
\]
which possesses the typical saddle path stability.

At every point below \( C(K,p_K) = 0 \), constraint (2) is violated so the system may never be located on this region once regulation is imposed. Our assumption of \( r^* < r_0 \) implies that the initial unregulated steady state is located below the locus \( C(K,p_K) = 0 \). Let \( K^* \) and \( p_K^* \) denote the regulated long run equilibrium values for \( K \) and \( p_K \) respectively. The above considerations entail that \( (K^*,p_K^*) \) is on the locus \( C(K,p_K) = 0 \). Furthermore, the optimal path approaching this equilibrium is that determined by the following system of equations:

\[
\begin{align*}
\dot{K} &= I(p_K) - \delta(s)K \\
\dot{p}_K &= p_K(r + \delta(s)(1 - \lambda) - \lambda \rho^*) - sR_s(1 - \lambda) \quad \text{and} \\
C(K, p_K) &= 0.
\end{align*}
\]

We have then, that \( K \) and \( p_K \) move along the constraint towards their regulated equilibrium values. The long run steady state \( (K^*,p_K^*) \), is obtained by setting \( \dot{K} = \dot{p}_K = 0 \) in (9). Thus, \( (K^*,p_K^*) \) is the intersection point of the schedules \( \dot{K} = 0 \) and \( C(K,p_K) = 0 \) yielding \( K^* > K_0 \). This result corroborates the existence of overcapitalization in the dynamic setup.

In order to compare the adjustment process towards a regulated equilibrium, assume that the initial unregulated steady state is shared by two systems: one with flexibility in the choice of \( s \) and the other one with a fixed \( s \), and hence \( \delta \). It is not hard to check that the presence of flexibility in the choice of \( s \) has the effect of rotating the locus \( \dot{K} = 0 \) clockwise around the initial steady state leaving the other loci unaffected. This fact yields a regulated equilibrium point with a higher level of capital and a lower level of investment, implied by a lower equilibrium \( p_K^* \), for the system with a variable rate of capital utilization. The firm thus offsets the higher marginal cost of investment by lowering the utilization rate on its capital stock, this allows it to achieve a higher equilibrium level of capital \( (K^*) \). These considerations are depicted geometrically in figure 1, where \( A \) stands for the initial unregulated equilibrium shared by both systems, \( B \) for the regulated equilibrium for the system with a constant \( s \), and \( C \) denotes the regulated equilibrium when there is flexibility in the choice of \( s \). To reach their regulated equilibrium points, both systems move to a point such as \( A' \), when regulation is imposed. Afterwards, they proceed along the constraint in the direction indicated by the arrows until they reach their regulated equilibrium.

In this model, the equilibrium rate of gross investment is given by replacement investment, \( \delta(s)K \), and a lower equilibrium investment rate than that of the unregulated equilibrium is feasible whenever the slope of the \( \dot{K} = 0 \) locus is negative. This holds whenever the partial derivative of total depreciation, \( \delta(s)K \), with respect to \( K \) is negative. Previous efforts to justify a lower investment rate within the regulated firm had to give up overcapitalization. Examples of such attempts may be found in Peterson and vander Weide (1975) and Dechert (1984).

The equilibrium value \( \lambda^* \) is given by

\[
\lambda^* = 1 - \frac{p_K^*(r - \rho^*)}{s^* R_s^* - p_K^*(\rho^* + \delta^*)} = 1 - \frac{(r - \rho^*)}{s^* \delta^*(s^*) - \delta^*(s^*) - \rho^*},
\]

We have then that \( K \) and \( p_K \) move along the constraint towards their regulated equilibrium values.
Given that the denominator in equation (11) is negative and that \( r^* > r \), we have that \( \lambda^* < l \) must hold. Combining this last relation with equations (6) and (7), we obtain that the steady state level of unit services (\( s^* \)) satisfies

\[
s^* \delta'(s^*) - \delta(s^*) = r - \frac{\lambda^*(r^*-r)}{(1-\lambda^*)} < r.
\]

The initial, unregulated, level of unit services \( s_0 \) is such that the equality \( s_0 \delta'(s_0) - \delta(s_0) = r \) holds; hence, due to the fact that the function \( s\delta' - \delta \) is increasing in \( s \), we have that the inequality \( s^* < s_0 \) is satisfied. We may interpret (12) in the following manner: the total cost of producing a given level of services (\( sK \)) at the long run steady state, is given by \( rK + \delta(s)K \), where the first term is the opportunity cost of holding capital and the second term is the cost due to depreciation. This cost is minimized when \( s \) is chosen so as to satisfy \( s\delta'(s) - \delta(s) = r \). Thus, (12) implies that total services are produced with an inefficient mix of \( s \) and \( K \): the level of unit services (or the utilization rate) is too low and the stock of capital too large.

The marginal rate of technical substitution between the long run values of capital and labor can be calculated from (5) and (7) as

\[
\frac{s^* F_{x^*}}{F_{L^*}} = \frac{p_{x^*}(r + \delta^*)}{w} - \frac{\lambda^* p_{x^*}(r^*-r)}{w(1-\lambda^*)}.
\]

Given the choice of \( r^* \), the second term on the right hand side is positive; thus, equation (11) depicts the usual A-J effect pertaining to an inefficiently high capital-labor ratio in order to produce a given level of output. The magnitude of \( \lambda^* \) is, as usual, proportional to the magnitude of the inefficiency.

### 3. THE IMPACT OF A VARIABLE RATE OF CAPITAL UTILIZATION

Assume now that a small change in the fair rate of return \( r^* \) is brought about by the regulator, say for instance that \( r^* \) is lowered. Performing a lengthy, but otherwise straightforward, comparative statics exercise on the equilibrium values of the variables, we obtain the following results:

I) More regulation, that is a lower \( r^* \), brings about a higher steady state level of capital; moreover, this overcapitalization is enhanced whenever the rate of capital utilization is variable,

II) The utilization rate \( s \) is lower when the fair rate of return \( r^* \) is lowered; in particular, the regulated monopoly exhibits a lower depreciation rate of its capital stock.

III) Total services \( S=sK \) increase when \( r^* \) is lowered, and this increase is mitigated by the presence of a variable \( s \).

IV) The magnitude of the multiplier \( \lambda^* \) increases as \( r^* \) is lowered; furthermore, this increase is mitigated when \( s \) is variable.

V) The effect on labor and output is ambiguous.

The first two results are consistent with those obtained in the previous section, the third one implies that “overcapitalization dominates underservicing” so that total services of capital increase given more regulation. As a consequence of IV), the presence of a variable utilization rate brings about a decrease in the magnitude of the inefficiency due to the high capital-labor
ratio implied by equation (13). This is caused by the lower opportunity cost of holding capital due to a reduction in the depreciation rate.

This last result could explain why empirical work related to the measurement of the A-J effect via the detection of an inefficiently high capital-labor ratio has had its share of problems. As mentioned in the introduction, it is this input bias that is searched for in most empirical studies. A variable rate of capital utilization may cause this inefficiency to be very small and thus hard to measure. This does not mean, though, that the A-J effect is not present, but rather that the inefficiency lies somewhere else; specifically, in the composition of total services as implied by (12). Our model suggests that the presence of the A-J effect could be tested by means of equation (12), or in other words, by searching for underservicing or underdepreciation of the capital stock within the regulated firm.

4. ANTICIPATED Deregulation

In order to take advantage of the new dynamic setup we analyze the following plausible situation:

Assume that the firm is resting at its long run regulated steady state (point C in figure 1), when at an initial time t = 0, the regulator unexpectedly announces that at a specified future time (t = τ), regulation will be completely lifted. How does the firm adjust to its new equilibrium? At first sight, the outcome looks grim: the firm would like to reduce its capital stock and avoid the losses caused by a fall in the price of capital (p_K) at t = τ: capital losses could take place at the initial time but then profits increase and the constraint is violated10. Clearly something is amiss in the above reasoning, specifically, we have not considered the possibility of inefficiency in either the choice of labor (L), or unit services (s). Let’s assume, without loss of generality, that R_L = w, but R_S ≠ p_s^* (in particular, λ = I) once the regulator’s announcement is made. The system is now determined by

\[ \dot{K} = I(p_K) - δK \]
\[ p_K = p_K(r - p^*) \] and
\[ C(K, p_K, s) = 0. \]

We may represent the constraint \( C(K, p_K, s) = 0 \) for any given level of unit services (\( s \)) on the K-p_K plane. When \( s \) is such that, \( R_s ≠ p_s^* \) it must lie below the constraint in figure 1, where unit services are efficiently chosen. The firm will then either “overutilize” or “underutilize” capital throughout the adjustment, allowing p_K to fall at t = 0 without violating the constraint. By doing so, the firm begins shedding its “excess” capital stock, and reaches the unregulated saddle path at t = τ avoiding capital losses at that time. During the adjustment, the system follows a path such as that labeled by a in figure 2. Here, A stands for the unregulated final equilibrium, B the initial regulated steady state, B’ the point that is reached at t = 0, when the regulator makes his announcement, and A’, attained at t = τ, is a point on the saddle path of the final deregulated system. The dynamics are those embodied by (14) below the constraint, and those of the unconstrained problem above it. The inefficient service market causes the constraint to move down throughout the adjustment period, as indicated by the arrows in figure 2.

What is interesting in the above analysis, is that the firm need not incur in inefficiencies within the labor market, as it would have to, when the utilization rate is not a variable subject to
When the system goes from an unregulated steady state towards a regulated one, this static inefficiency is not called for, since the initial capital gain can take place without violating any constraint. Here, in the opposite situation, we must have this static inefficiency all throughout the adjustment in order for dynamic efficiency to be achieved.

5. CONCLUSIONS

Within the partial equilibrium context, specifically at the firm’s level, we analyzed the effect that an endogenous utilization rate of capital had on a firm subject to a rate of return constraint. We found out that this type of regulation causes a reduction in the optimal utilization rate, due to its impact on the net real interest rate as perceived by the firm. Furthermore, the total flow of services of capital was produced with an inefficient mix of unit services and capital. The well-known Averch-Johnson effect of overcapitalization was magnified by the presence of a variable utilization rate. The firm reduces the utilization rate of capital when regulation is introduced; as a consequence, the inefficiency associated with a high capital-labor ratio was mitigated. We concluded that existing empirical studies would have trouble detecting the Averch-Johnson effect when trying to measure an inefficiently high capital-labor ratio. We suggested that the “true” inefficiency lies in the production of total services of capital \( S \); hence, a possible test of the Averch-Johnson effect could be obtained by searching for “underservicing”, or a lower depreciation rate within the regulated firm.

In our model, it was possible to have a regulated firm with a lower equilibrium level of investment than the one it would have without regulation. This last result cannot be obtained without assuming variability of services unless the Averch-Johnson overcapitalization effect is given up. In contrast, here it was possible for overcapitalization to coexist with a lower investment rate. Finally, we studied the impact of anticipated deregulation on a currently regulated firm. We found that the firm could use the variable utilization rate in order to, either “overdepreciate” or “underdepreciate” the capital stock during the adjustment period. This had the double purpose of first, avoiding the violation of the rate of return constraint while it was still in place and second, of not having to resort to labor market inefficiencies during the transition period towards the unregulated equilibrium. We did not pursue this model any further, since it was not our intention to get deeply involved in the regulatory process; however, we think further research is called for, particularly in the area of anticipated deregulation.

ENDNOTES

1 If for example, the firm were a revenue or an output maximizer, the effect the regulatory constraint would be to cause undercapitalization.
3 This last assumption does not allow the firm to adjust instantaneously to its desired capital stock so the dynamics of the problem become relevant. An alternative way of proceeding is to introduce adjustment costs for investment. The results are completely analogous; the monopsony assumption was adopted for ease of notation.
4 Strict concavity yields \( R > sKR_S + LR_L \), so combining this expression with (2) and (5) we obtain the desired result.
5 If the initial capital stock is different from \( K_0 \), then the optimal path towards the regulated steady state may include portions where constraint (2) is not binding.
6 As mentioned in El-Hodiri and Takayama (1981), this is not entirely obvious since the marginal cost of investment will also be higher.
7 This could happen due to the lower utilization rate $s$ and hence depreciation, brought about by a higher capital stock. The convexity of the function $\delta$ is required.
8 An exception is Nemoto, Nakanishi and Madono (1993), who avoid estimating the multiplier $\lambda$ and measure overcapitalization directly.
9 Even though this setup seems quite natural, we have not found any dynamic treatments of anticipate deregulation.
10 Anticipated regulation does not pose this problem since the constraint is not symmetric.
11 The term inefficient may be misleading in this context given that the firm is still choosing the optimal path in order to maximize lifetime profits. Here, it may do so only by incurring in static inefficiencies.
12 The firm could of course, choose to have an inefficient labor market instead, what is not possible is for the inefficiency to exist in both services and labor since the system would be indeterminate.

REFERENCES


